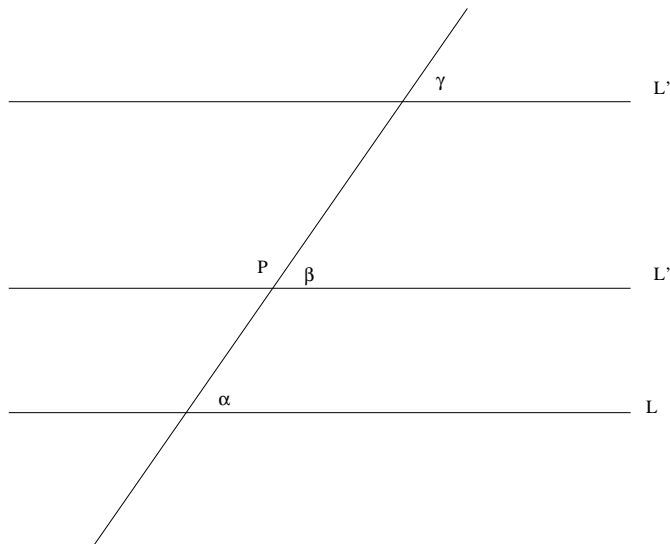


MATHEMATICS 3100. First Midterm Test: Solutions.

February 6, 2003

1. [20 points] Suppose that line L is parallel to line L' and the line L' is parallel to the line L'' . Show that the line L is parallel to L'' .

Solution. See I12, # 1. Here is an alternative solution. Draw the line M transversal to L and L' . See Figure . The line M cannot be parallel to L'' since otherwise there would be two distinct lines M, L' through the point P which are parallel to L'' . Hence M is also transversal to L'' . Now by the principle of corresponding angles, $\alpha = \beta, \beta = \gamma$. Therefore $\alpha = \gamma$ and by the converse to the principle of corresponding angles, $L \parallel L''$. \square



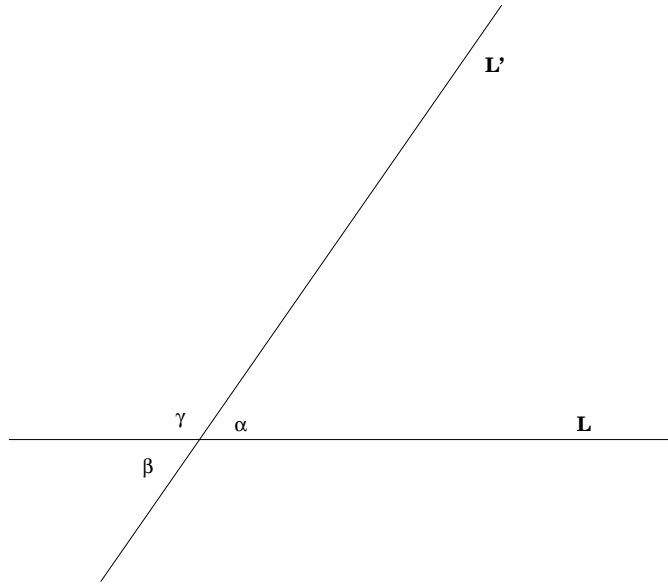
2. [20 points] State and prove the principle of vertical angles.

Solution.

Principle of vertical angles. *If the lines L and L' cross then the opposite angles α and β have equal measures.*

Proof Introduce the angle γ as in the Figure . Then $\alpha + \gamma = \pi, \beta + \gamma = \pi$ because L and L' are straight lines. Hence

$$\alpha + \gamma = \beta + \gamma \Rightarrow \alpha = \beta. \quad \square$$

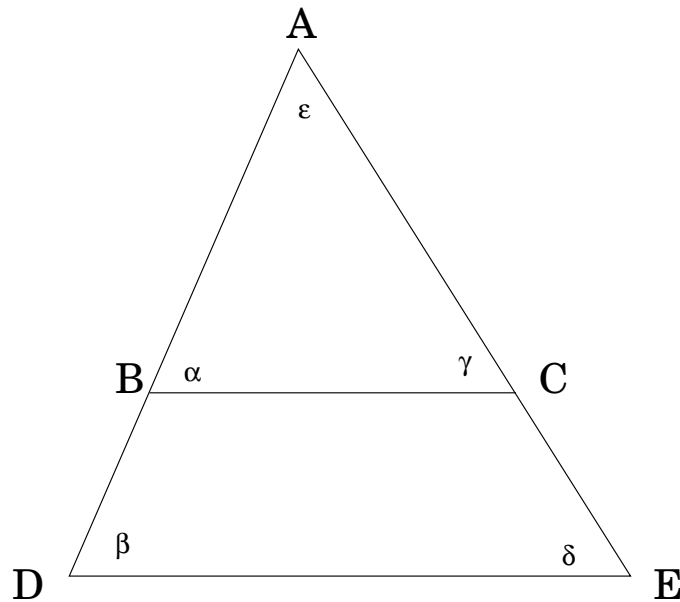


3. [20 points] Prove that the function $f(x, y) = (x + a, y + b)$ is a motion of the plane.

Solution. See theorem 9, page 211. □

4. [20 points] If BC is parallel to DE show that the triangles $\triangle ABC$ and $\triangle ADE$ are similar.

Solution. Since BC is parallel to DE , by the principle of corresponding angles $\alpha = \beta, \gamma = \delta$. Hence the corresponding angles of the triangles $\triangle ABC$ and $\triangle ADE$ are the same: α, γ, ϵ . Therefore by the AAA principle, $\triangle ABC$ and $\triangle ADE$ are similar. □



5. [20 points] Prove that if m is a motion of the line such that $m(0) = 0$ then either $m(x) = x$ or $m(x) = -x$.

Solution. See theorem 3, page 201.

□