MATH. Third Midterm Test: Sample.

April 15, 2003

The exam is “closed book, closed notes”.

1. [20 points] Given points \( P = (1, 2) \) and \( Q = (1, -1) \) find a motion \( m \) of the plane so that \( m(P) = (0, 0) \) and \( m(Q) = (d, 0) \) for some \( d > 0 \).

2. [20 points] Using axioms of the plane show that given a pair of distinct points \( P, Q \) in the plane there exists a straight line \( L \) through these points.

3. [20 points] Suppose that \( P \) is an \( n \)-gon in the plane so that \( n - 1 \) perpendicular bisectors \( L_1, \ldots, L_{n-1} \) for the sides of \( P \) cross at the same point. Show that the vertices of \( P \) belong to the same circle.

4. [20 points] State and prove the triangle inequality in the plane.

5. [20 points] Suppose that \( P \) is a pyramid whose base is the circle, height of \( P \) equals \( h \) and the angle between the axis of the pyramid and its surface equals \( \alpha \). See figure below. Compute the volume of \( P \). Justify your computation!
\[ v \]
\[ \alpha \]
\[ h \]
\[ O \]