MATHEMATICS 3220-1. Third Midterm Test (Sample).

April 28, 2002

The exam is "closed book, closed notes". All problems should be treated as problems about "proofs"; just the correct computation without proper justification can result in a very low score on the problem.

1. [15 points] State the theorem which guarantees differentiability of a function $f : \mathbb{R}^n \to \mathbb{R}^m$ in terms of the partial derivatives $\frac{\partial f_i}{\partial x_j}$.

2. [15 points] Write Taylor's formula for the function $f(x, y) = x^3 y$ for $p = 3$ at $\mathbf{a} = (0, 1)$.

3. [20 points] State the definition of a convex set and prove that the set

$$E = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$$

is convex.

4. [15 points] Suppose that $f : \mathbb{R} \to \mathbb{R}^2$ and $g : \mathbb{R}^2 \to \mathbb{R}$ are differentiable everywhere, $f(0) = (0, 0)$, $f'(0) = (1, 2)$ and $\nabla g(0, 0) = (2, 3)$. Let $h = g \circ f$. Compute $h'(0)$.

5. [15 points] For the following function prove that f^{-1} exists and is differentiable in some nonempty open set containing $\mathbf{a}$ and compute $D(f^{-1})(\mathbf{a})$:

$$f(u, v) = (3u - v, 2u + 5v), \mathbf{a} = (1, 1).$$

6. [20 points] State and prove the theorem about differentiability and the total derivative of the sum of two vector-valued functions. (You can use without a proof the basic lemma with the equivalent definition of differentiability.)