

MATHEMATICS 3220-1. Second Midterm Test (Sample).

March 26, 2002

The exam is “closed book, closed notes”. All problems should be treated as problems about “proofs”; just the correct computation without proper justification can result in a very low score on the problem.

1. [15 points] Evaluate

$$\frac{d}{dx} \int_{-1}^1 \sqrt{x^2 + y^2 + 1} dy, \quad \text{at } x = 0.$$

2. [15 points] Find the interior of the set $\{(x, y) : x = y^2\}$.

3. [20 points] Prove the theorem on sequential compactness of closed and bounded subsets of \mathbb{R}^n .

4. [15 points] Let $E = \{(x, y) : x^2 + y^2 < 4\}$. Determine if the subset $A \subset E$, $A = \{(x, 0) : -2 < x < 2\}$, is relatively closed, relatively open or neither.

5. [15 points] State the definition of a connected subset of \mathbb{R}^n .

6. [20 points] Prove that the circle

$$\{(x, y) : x^2 + y^2 = 1\}$$

is connected.