Name:

Student ID number:

Alias:

MATH 2270-2. Final Test: Sample.

You have 120 minutes for this test. The exam is “closed book and closed notes” no calculators. You can use “cheat sheets”. Show all your work. Correct answers with incorrect justification may result in a zero score on the problem. If this or something else is unclear, ask me!

An alias is any combination of letters and numbers, between 2 and 6 symbols. I will use it to post the scores on the door of my office.

1	2	3	4	5	6	7	8

Total:

Grade:

Problem 1. (10 points) Find coordinates of the vector $\overrightarrow{v} = (2, 2)$ with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$$

in $\mathbb{R}^2$.

Problem 2. (15 points) Let S consist of all vectors in $\mathbb{R}^3$ with integer coordinates (i.e., we allow coordinates to be $0, \pm 1, \pm 2, \ldots$, but for instance $\sqrt{2}$ is not allowed). Determine whether or not the subset S is a subspace in V. Justify your answer!

Problem 3. (15 points) Using standard basis in P_2 find matrix representation, and eigenvalues of the linear transformation $T : P_2 \to P_2$ which is given by the formula:

$$T(p(x)) = p(1) + p(x-1) + p'(x).$$

Problem 4. (10 points) Write the augmented matrix and find all solutions of the linear system (in 5 unknowns):

$$\begin{cases} x_2 + x_3 + x_4 + x_5 = 1 \\ x_1 + x_3 = 0 \\ x_1 + x_2 - x_4 = 1 \end{cases}$$

Problem 5. (15 points) Find an orthonormal basis in the subspace V in $\mathbb{R}^3$ spanned by the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 6. (15 points) Find all eigenvalues of the matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

For each eigenvalue find a basis of the corresponding eigenspace. Determine if the matrix is diagonalizable.

Problem 7. (10 points) Suppose that $1, 2, 3$ are the eigenvalue of the 3×3 matrix A. Determine what are the eigenvalues of the matrix A^2.

Problem 8. (10 points) Decide if the quadratic form $q(x, y) = x^2 + y^2 - xy$ is positive-definite or indefinite. Justify your anwer.