MATHEMATICS 2270-1. First Midterm Test (Sample).

October 4, 2002

You have 90 minutes for this test. No calculators, no textbooks, no notes are allowed. You can use "cheat sheets".

1. Find all solutions of the linear system

$$\begin{cases} x_3 + x_4 + x_5 + x_6 = 0 \\ x_1 + x_2 - 2x_6 = 0 \\ x_2 + x_4 - x_5 = 0 \end{cases}$$

2. Using the row echelon reduction find inverse of the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$$

3. Determine if the following vectors are linearly independent:

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$$

4. Find basis of the image of the linear transformation

$$T(\vec{x}) = \begin{pmatrix} x - y + 2z \\ x - y + 2z \\ x - y + 2z \end{pmatrix}, \quad \text{where} \quad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

5. Find an orthonormal basis in the subspace V in $\mathbb{R}^4$ spanned by the vectors

$$\begin{pmatrix} 1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 6 \\ -4 \\ 6 \end{pmatrix}.$$

6. Compute the following determinant using the definition of the 3×3 determinant:

$$\begin{vmatrix} 1 & 1 & 2 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{vmatrix}.$$