

MATHEMATICS 2270. Homework # 9.

1. Consider the vector space

$$V = \text{Span}\{\sin(t), \cos(t)\}$$

with the basis: $\mathcal{B} = \{f(t) = \sin(t) - \cos(t), g(t) = \sin(t) + 2\cos(t)\}$. Compute the coordinates of the function $2\sin(t) - \cos(t)$ with respect to the basis \mathcal{B} .

2. Determine whether or not the following collection of vectors spans P_2 :

$$\{p_1(t) = t^2 + 1, p_2(t) = 2t + 1, p_3(t) = t^2 + 2t + 2, p_4(t) = -t^2 + 2t\}.$$

(Hint: first find dimension of the space $\text{Span}\{p_1(t), p_2(t), p_3(t), p_4(t)\}$.)

3. Find matrix representation, rank and nullity of the linear transformation $T : P_1 \rightarrow P_2$ which is given by the formula:

$$T(p(x)) = xp(x)$$

Here P_1 is the space of polynomials $b_0 + b_1x$ of degree ≤ 1 (with the standard basis $\{1, x\}$) and P_2 is the space of polynomials $a_0 + a_1x + a_2x^2$ of the degree at most 2 with the standard basis $\{1, x, x^2\}$.

4. §4.3, # 2.
5. §5.5, # 6.
6. §5.5, # 9.