Name:
Password:

Math 3210-2. Final Test (Sample).

Do eight of the following problems. If you do more then I will grade only the first eight problems that you did. Each problem is worth 25 points. The exam is "closed book and closed notes". In your solutions you can use formulas for the derivatives of $e^x, \log(x)$ (the latter is the natural logarithm), of the trigonomentric functions and of the polynomial functions. You can also use that these functions are continuous on their domains. All solutions should be considered as proofs (even if in the problem I am asking you to compute something), correct answers with incorrect proof may result in a zero score on the problem (depending on how far your proof is from being correct). If this or something else is unclear, ask me!

1	2	3	4	5	
6	7	8	9	10	Total Score

1. State (write down to the best of your abilities) the Completeness Axiom for the real numbers.

2. (a) Prove that if $(x_n), (y_n)$ are convergent sequences of real numbers then

$$\lim_{n\to\infty} (x_n + y_n) = \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n.$$

(b) Give an example of the sequences $(x_n), (y_n)$ of real numbers such that neither (x_n), nor (y_n) converges, but the sequence $(x_n + y_n)$ converges.

3. For the function $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$$

compute the image $f(E)$ of the set $E = [-2, 3]$. As in other problems you can use the fact that polynomial functions are continuous on $\mathbb{R}$.

4. Compute the limit (or show that it does not exist)

$$\lim_{x\to 0} x\cos(\frac{1}{x}).$$

You can use limit theorems.

5. Show that for each real number $a \in [0, 1)$,

$$\lim_{n \to \infty} a^n = 0.$$

DO NOT USE LOGARITHMS!

6. Using the definition of the limit compute

$$\lim_{x \to 0} \frac{1}{x^2}.$$

(Here you can first "guess" what the limit is, but then you would have to give a proof that your answer is correct.)

7. Prove the following inequality for all $x \geq 1$:

$$e^{x-1} > \log(x).$$

8. Prove that Dirichlet's function $f(x) = 0, x \notin \mathbb{Q}$, $f(x) = 1, x \in \mathbb{Q}$, is not integrable on $[0, 1]$.

9. State (write down to the best of your abilities) the Bolzano-Weierstrass Theorem.

10. Compute the derivative $f'(0)$ for the function

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \geq 0 \\ \cos(x) & \text{if } x < 0 \end{cases}$$

or show that this derivative does not exist.