

MATHEMATICS 3210-1. 3-rd Midterm Test (Sample).

November 10, 2001

The exam is “closed book and closed notes”. In your solutions you can use formulas for the derivatives of e^x , $\log(x)$, of the trigonometric functions and of the polynomial functions. You can use that these functions are continuous on their domains.

- (20 points) For $L, a \in \mathbb{R}$ prove the following:
 $\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = L$ if and only if $\lim_{x \rightarrow a} f(x) = L$.
You can use ϵ - δ definition of the limit if you like.
- (15 points) Give the definition of
 - Limit $\lim_{x \rightarrow -\infty} f(x) = L$.
 - The derivative of a function f at $a \in \mathbb{R}$.
- (15 points) Prove that there exists $x \in \mathbb{R}$ such that $\frac{e^x - e^{-x}}{2} = \cos(x)$.
- (15 points) Compute the limit (or show that it does not exist)

$$\lim_{x \rightarrow 0^+} x \cos\left(\frac{1}{x}\right).$$

5. (20 points) Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a strictly increasing, everywhere differentiable function. Is it true that $f'(x) \geq 0$ for each $x \in \mathbb{R}$? Give a proof!

- (15 points) Determine if the series

$$\sum \frac{1}{n \log(n)}$$

converges.