MATHEMATICS 3210-1. Second Midterm Test (Sample).

October 17, 2001

The exam is "closed book, closed notes", you can use a 1 page cheat sheet with the elementary college algebra formulae. All problems should be treated as problems about "proofs"; just the correct computation without proper justification can result in a very low score on the problem.

1. (15 points) Prove or disprove the following:
For each real number x there exists a natural number n such that $n/(n^2 - n) > x$.

2. (15 points) State the Bolzano-Weierstrass Theorem.

3. (15 points) Compute the limit (or show that it does not exist)

$$\lim_{n\to\infty} \frac{\sqrt{n} - n}{\sqrt{n} + 2n}$$

(You can use Limit Theorems.)

4. (20 points) Let $x_1 \geq 3$. Define inductively the sequence (x_n) as $x_{n+1} = \sqrt{3x_n}$. Show that this sequence is decreasing. Find the limit of this sequence.

5. (20 points) State and prove the theorem about the limit of sum of two convergent sequences.

6. (15 points) Determine weather or not the following series converges:

$$\sum_{i=1}^{\infty} \frac{\sin(n) + \cos(n)}{3^n}.$$

(You can use tests for convergence.)