MATHEMATICS 3210-1. First Midterm Test (Sample).

September 11, 2001

1. Using axioms of real numbers only show that product of 0 and every real number equals zero.

2. State the completeness axiom for the real numbers.

3. Prove or disprove that the sets

$$A = \{n \in \mathbb{Z} | n = -5k + 1 \text{ for some } k \in \mathbb{N}\},$$

and $\mathbb{N}$ have the same cardinality.

4. Represent the function

$$f(x) = 1 + x^2 + x^4 + x^{2/3}$$

as a nontrivial composition of two functions.

5. Negate the following statement:
"For each $x \in \mathbb{R}$ there exists a natural number n so that either $nx > 1$ or $nx < -1$."

6. Show that for each $n \in \mathbb{N}$

$$n! \geq 2^n - 2.$$