

MATHEMATICS 3210-1. Homework 5.

October 2, 2001

1. Define a sequence (x_n) inductively by starting with $x_1 \geq 2$ and taking $x_{n+1} = 1 + \sqrt{x_n - 1}$. Show that x_n is nonincreasing.

What happens if you start with $1 \leq x_1 < 2$?

2. Find the limit of each of the following sequences using limit theorems:

(a) $x_n = (1 + n - 3n^2)/(3 - 2n + n^2)$.

(b) $x_n = (n^3 + n - 5)/(5n^3 + n - 1)$.

(c) $x_n = \sqrt{2n^2 - 1}/(n + 1)$.

3. Prove or disprove the following:

(a) For each $C > 0$ there exists a number $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ ($n \in \mathbb{N}$) one has

$$n^2 - n + 2 \geq C.$$

(b) For each $C \in \mathbb{R}$ there exists a number $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ ($n \in \mathbb{N}$) one has

$$-\sqrt{n} + \cos(n) + (-1)^n \leq C.$$

(c) For every $C \in \mathbb{R}$ there exists $n_0 \in \mathbb{Z}$ so that for all integers n satisfying $n \leq n_0$, one has

$$-n^2 - 2n + 1 \leq C.$$

4. Let $x_1 \geq 3$. Define inductively the sequence (x_n) as $x_{n+1} = \sqrt{3x_n}$. Show that this sequence is decreasing.