

# MATHEMATICS 3210-2. Homework 2.

September 4, 2001

1. Show that  $\mathbb{Z}$  (the set of all integers,  $\mathbb{Z} = \{\dots, -1, 0, 1, 2, \dots\}$ ) is countable.

2. Represent the function

$$f(x) = \cos(x^2 + \log(x)) + (x^2 + \log(x))^2$$

as a nontrivial composition of two functions. Here  $\log(x)$  is the natural logarithm.

3. Using induction prove that:

(a)  $n < 2^n$ ,

(b)  $n^2 \leq 2^n + 1$ ,

for all natural numbers  $n = 1, 2, 3, \dots$

4. Prove that sum of rational and irrational number is always irrational.

What about the product rational·irrational? When is it rational and when it is irrational?