

MATHEMATICS 3210. Solutions for Homework # 1.

1. Write the negation of each statement:

- (a) M is a symmetric matrix.
- (b) Today is Wednesday or it is snowing.
- (c) Bob and Betty are related.
- (d) If it rains today, then the roof will leak.
- (e) If roses are red and violets are blue, then I love you.

Solution. (a) First, let's expand the statement " M is a symmetric matrix":

"The number of rows in M is the same as the number of columns
(A)

AND

for each pair of indices i, j we have $m_{ij} = m_{ji}$ (B)".

(Note the universal quantifier in (B)!)

Thus the negation of the above statement has the form "NOT (A)
OR NOT B".

The negation of (A) is "The number of columns is different from the number of rows", the negation of (B) is "There exists a pair of indices (i, j) such that $m_{ij} \neq m_{ji}$ ". Hence the negation of the entire statement is:

"The number of columns is different from the number of rows AND there exists a pair of indices (i, j) such that $m_{ij} \neq m_{ji}$ ".

(b) "Today is not Wednesday AND it is now snowing".

(c) This is a trick question, note that AND in the above sentence does not connect statements, it connects names. Hence the negation is

"Bob and Betty are not related" (equivalently, "Bob and Betty are unrelated").

(d) The statement has the form of the implication $A \Rightarrow B$. The only case when the implication is false is when the assumption is true but the conclusion is false. Hence the negation is

"It rains today AND the roof does not leak" (equivalently, "It rains today but the roof does not leak").

(e) The statement is again implication $(A \& B) \Rightarrow C$, where A: "The roses are red", B: "The violets are blue", C: "I love you".

Hence the negation is " $(A \& B)$ AND NOT C ". Hence the negation of the entire statement is

"The roses are red and violets are blue AND I do not love you."

2. Show that for any sets B, A_1, A_2 we have $B \setminus (A_1 \cup A_2) = (B \setminus A_1) \cap (B \setminus A_2)$.

3. Problem # 1 from the handout. For $f : A \rightarrow B$, compute $f(A_0), f^{-1}(B_0)$.

(a) $f(x) = x^2, f : A = \{-1, 0, 1\} \rightarrow B = \mathbb{R}. A_0 = \{-1, 1\}, B_0 = \{0, 1\}$.

(b) $A = B = \mathbb{R}$, $A_0 = \{x \in \mathbb{R} | x > 0\}$, $B_0 = \{0\}$,

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$$

(c) $A = B = \mathbb{R}$, $A_0 = B_0 = (-2, 1) = \{x \in \mathbb{R} | -2 < x < 1\}$,

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$