MATHEMATICS 2210-3. Homework 2.

January 17, 2001

1. Problems # 2 (c,e), 5 (a), 18 from Section 13.3.

Problem # 2 (Each item is worth 5 points). Let $\vec{a} = (3, -1)$, $\vec{b} = (1, -1)$, $\vec{c} = (0, 5)$. Find each fo the following:

(c). $(\vec{a} + \vec{b}) \cdot \vec{c}$.

Solution. $\vec{v} = \vec{a} + \vec{b} = (3, -1) + (1, -1) = (4, -2)$. $\vec{v} \cdot \vec{c} = (4, -2) \cdot (0, 5) = -10$. Hence $(\vec{a} + \vec{b}) \cdot \vec{c} = -10$.

(e). $|\vec{b}|\vec{b} \cdot \vec{a}$.

Solution. $\vec{b} \cdot \vec{a} = (1, -1) \cdot (3, -1) = 3 + 1 = 4$. $|\vec{b}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$. Hence $|\vec{b}|\vec{b} \cdot \vec{a} = 4\sqrt{2} = \sqrt{32}$.

Problem # 5 (a). (5 points) Write the vector $\overrightarrow{AB}$ in the form $a_1 \vec{i} + a_2 \vec{j}$. Here A has coordinates $(2, 2)$, and B has coordinates $(-3, 4)$.

Solution. $\overrightarrow{AB} = (-3, 4) - (2, 2) = (-5, 2) = -5\vec{i} + 2\vec{j}$.

Problem # 18. (5 points) For what numbers c are $2c\vec{i} - 8\vec{j}$ and $3\vec{i} + c\vec{j}$ orthogonal?

Solution. $(2c\vec{i} - 8\vec{j}) \cdot (3\vec{i} + c\vec{j}) = 6c - 8c = -2c$. The vectors are orthogonal if and only if their dot product is zero. Hence $-2c = 0$, and thus $c = 0$.

2. Problems # 24, 32 from Section 13.4. Each problem is worth 10 points.

Problem # 24. The position of a moving particle at time t is given by $\overrightarrow{r(t)}$. Find the velocity and acceleration vectors $\vec{v}$ and $\vec{a}$ and speed at the time $t_1 = 1/2$. Sketch a portion of the graph of $\overrightarrow{r(t)}$ containing P with $\overrightarrow{OP} = \overrightarrow{r(t_1)}$ and sketch $\overrightarrow{v(t_1)}, \overrightarrow{a(t_1)}$ with their initial points at P.

Here $\overrightarrow{r(t)} = (3t^2 - 1)\vec{i} + t\vec{j}$.

Solution. $\overrightarrow{v(t)} = \frac{d}{dt}\overrightarrow{r(t)} = 6t\vec{i} + \vec{j}$, $\overrightarrow{a(t)} = \frac{d}{dt}\overrightarrow{v(t)} = 6\vec{i}$. At $t_1 = 1/2$ we get:

$$\overrightarrow{v(0.5)} = 3\vec{i} + \vec{j}, \overrightarrow{a(0.5)} = 6\vec{i}.$$

Speed equals $|\overrightarrow{v(0.5)}| = \sqrt{9+1} = \sqrt{10}$. See Figure 1.

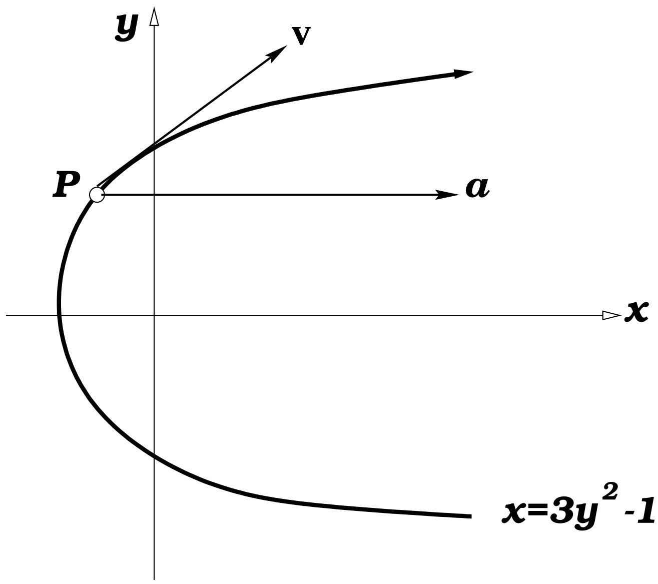

Figure 1: *Problem 24.*

Problem # 32. Find the velocity vector $\overrightarrow{v(t)}$ and the position vector $\overrightarrow{r(t)}$ given that

$$\overrightarrow{a(t)} = t\vec{j}, \overrightarrow{v(0)} = \vec{i} + 2\vec{j}, \vec{r}(0) = \vec{0}.$$

Solution. $\overrightarrow{v(t)} = \int_0^t \overrightarrow{a(t)}dt = \int_0^t t\vec{j}\,dt = c\vec{i} + d\vec{j} + t^2/2\vec{j}$. Here c, d are scalars to be computed. Note that we have two unknown scalars since our vectors have two components. Since $\overrightarrow{v(0)} = \vec{i} + 2\vec{j}$, we get: $c = 1$, $d = 2$. Hence $\overrightarrow{v(t)} = \vec{i} + (2 + t^2/2)\vec{j}$.

Similarly,

$$\overrightarrow{r(t)} = \int_0^t \overrightarrow{v(t)}dt = t\vec{i} + (2t + t^3/6)\vec{j} + a\vec{i} + b\vec{j}.$$

To find a, b we use that $\vec{r}(0) = \vec{0}$ and hence $a = 0, b = 0$. Thus

$$\overrightarrow{r(t)} = t\vec{i} + (2t + t^3/6)\vec{j}.$$