Show all your work and make sure you justify all your answers.
1. These problems involve derivatives

(a) State the product rule.

(b) Differentiate $f(x) = \cos(x) \cdot (x^2 + 1)$

(c) Differentiate $f(x) = \cos(x) \cdot (x^2 + 1) \cdot \sin(x)$

(d) State the chain rule.

(e) Differentiate $f(x) = \cos((x^2 + 1) \cdot \sin(x))$

(f) Differentiate $f(x) = \sin(\cos(\sin^2(x)) \cdot (x^2 + 1) \cdot \sin(x))$

(g) State the quotient rule.

(h) Differentiate $f(x) = \frac{\cos(\sin(x^2 + 1) \cdot x^2)}{\cos(x)}$
2. These problems involve implicit derivatives. Find $\frac{dy}{dx}$ for the following equations.

(a) $\tan(\cos(\sin(x)y^3)) \cdot (y^4 + 1)^4 + x = 0$

(b) $(xy)^5 + \sec(xy) + x(y^6 + 1)^5 = 0$

(c) $x^3 + y^3 + x^2 y + y^2 x = (x^2 + y^2)^3$
3. This question will be on related rates and the test problem will be one the homework problems which I assigned. Also note that we discussed the solutions to all of these problems in class.

4. This problem will be on differentials and approximations. Define what a differential is, Study example 3 on page 144, problem 26, 23,17. The test problem will be extremely similar to one of two of these problems.

5. Find the maximum and minimum value of each of the following functions if they exist. If they exist you must state why it exists and if it does not exist you must state why it does not exist.

(a) \( f(x) = \frac{x^2 + x - 2}{x^2 + 1} \) on \([-1, 1]\)

(b) \( f(x) = \frac{-2}{x-4} \) on \([0, 1]\)

(c) \( f(x) = \cos(x) \) on \([-2, 1]\)

(d) \( f(x) = x^2 + 4x + 4 \) on \([-10, 10]\)

(e) \( f(x) = \frac{1}{x} \) on \([-1, 1]\)
6. Find where the functions are increasing, decreasing, concave up and concave down.

(a) \( f(x) = x^3 + 3x^2 - 12 \)

(b) \( f(x) = x^3 + 12x \)
7. Prove that if \( \frac{df}{dx} \) exists and is continuous on the interval I, and if \( \frac{df}{dx} \) is nonzero for all points \( x \) in I then either \( f(x) \) is increasing or decreasing on I. (Hint what does the intermediate value theorem say?)