

Name _____

Student ID # _____

Class Section _____

Instructor _____

Math 1210
Spring 2007

EXAM 1

Dept. Use Only Exam Scores		
Problem	Points	Score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
	TOTAL	

Show all your work and make sure you justify all your answers.

Math 1210
Exam

1. these problems involve the $\epsilon \delta$ definition of limit.

(a) State the $\epsilon \delta$ definition of $\lim_{x \rightarrow c} f(x)$.

(b) Use the definition that you gave in (a) to show that $\lim_{x \rightarrow 2} x^2 - 4 = 0$

(c) Use the definition that you gave in (a) to show that $\lim_{x \rightarrow c} ax + b = ac + b$

(a) let $F(x)$ and $G(x)$ be functions such that $0 \leq F(x) \leq G(x)$ for all x near c except possibly at c . Prove that if $\lim_{x \rightarrow c} G(x) = 0$ then $\lim_{x \rightarrow c} F(x) = 0$. (To get full credit you must use the definition given in 1 (a))

2. Compute six of the seven limits below. (If you do not clearly indicate which three you want graded I reserve the right to give you a zero score.)

(a) $\lim_{x \rightarrow 1} \frac{x^2+x-2}{x^2-1}$

(b) $\lim_{x \rightarrow 1} 2x(x^2 - 1)^{\frac{1}{2}}$

(c) $\lim_{x \rightarrow 1} \frac{x^2-1}{x^2+1}$

(d) $\lim_{x \rightarrow -1} \frac{x^2+x}{x^2-1}$

(e) $\lim_{x \rightarrow 0} \frac{1-\cos(x)}{\sin(x)}$

(f) $\lim_{x \rightarrow 0} \frac{\sin(x)^2}{x^2}$

(g) $\lim_{x \rightarrow \pi/4} x \tan(x)$

3. Compute six of the seven limits below. (If you do not clearly indicate which three you want graded I reserve the right to give you a zero score.)

(a) $\lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{x^2 - 1}$

(b) $\lim_{x \rightarrow 4} \frac{-2}{x - 4}$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x^2 + 1}$

(d) $\lim_{x \rightarrow \infty} x - (x^2 - 1)^{\frac{1}{2}}$

(d) $\lim_{x \rightarrow \infty} \frac{x^3 + x}{x^6 + 1}$

(f) $\lim_{x \rightarrow 0} \frac{\sin(x)^2}{x^2}$

(g) $\lim_{x \rightarrow \infty} \frac{\cos(x)^2}{x^2}$

4. These problems involve continuity.

(a) State the three criterion for a function to be continuous at a point c .

(b) Where is the following function continuous $f(x) = \frac{x-3}{x^2-9}$. What types of discontinuity does the function have and where?

(c) How would you define the function $x^4 \cos(1/x)$ so that it is continuous at $x = 0$?

(d) Show that the function $\frac{x-5}{|x-5|}$ has a non removable discontinuity at $x=5$.

5. These problems involve the intermediate value theorem.

(a) State the the intermediate value theorem.

(b) Use the intermediate value theorem to show that the equation $100x^9 - 5x^5 + 1 + \cos(x) = 0$ has a real solution.

(c) Prove that if $f(x)$ is continuous on $[0, 1]$ and satisfies $0 \leq f(x) \leq 1$ then $f(x)$ has a fixed point (ie there is a number c in $[0, 1]$ such that $f(c) = c$.) Hint consider the function $g(x) = x - f(x)$ and apply the intermediate value theorem.

6. Suppose that an object is dropped from a cliff and its position function is given by $p(t) = -16t^2 + 500$ find the velocity of the object.