Answer Key - practice problems

1. C
2. D
3. D
4. D
5. B
6. D
7. B
8. B
9. C
10. D
11. B
12. A
13. A
14. 36.6%
15. 47.5%
16. B
17. D
18. D
19. B
20. B
21. D
22.
23.
24. 47.5.
25. A
26. 38.5 years.
27. D

22. Let us first answer the first part of the question. The sample space here is a sequence of three tosses, each toss could give a head or a tail. So there are a total of $2 \times 2 \times 2 = 8$ elements in the sample space, each of which are equally likely. So probability of each outcome in the sample space is $\frac{1}{8}$. We can get 2heads in three tosses for the outcomes (H,H,T), (H,T,H), (T,H,H). So the probability of getting 2 heads in 3 tosses is $\frac{3}{8}$.
For the second part of the question, the sample space is a sequence of 2 tosses. There are 4 possible outcomes in the sample space, each of which has probability $\frac{1}{4}$. And there is only outcome where the first toss is a H and the second is T. So the probability of this event is $\frac{1}{4}$.

23. Here height is the explanatory variable x and weight is the response variable y. First let us find the standard deviation of x, s_x. The mean $\bar{x} = \frac{6+5+5.5+6.1}{4} = 5.65$. Thus the variance

$$s_x^2 = \frac{1}{3}\left[(6-5.65)^2 + (5-5.65)^2 + (5.5-5.65)^2 + (6.1-5.65)^2\right] = 0.257$$

The standard deviation s_x is thus $\sqrt{0.257} = 0.507$. The mean $\bar{y} = \frac{180+160+170+180}{4} = 172.5$. The standard deviation of y is

$$s_y = \sqrt{\frac{1}{3}\left[(180-172.5)^2 + (160-172.5)^2 + (170-172.5)^2 + (180-172.5)^2\right]} = 9.57$$

We next compute the correlation coefficient between x and y. It is

$$r = \frac{1}{3}\left[\frac{6-5.65}{0.507}\cdot\frac{180-172.5}{9.57} + \frac{5-5.65}{0.507}\cdot\frac{160-172.5}{9.57} + \frac{5.5-5.65}{0.507}\cdot\frac{170-172.5}{9.57} + \frac{6.1-5.65}{0.507}\cdot\frac{180-172.5}{9.57}\right] = 0.996$$

Thus the slope of the regression line is

$$b = r\frac{s_y}{s_x} = 18.8$$

The intercept of the regression line is

$$a = \bar{y} - b\bar{x} = 172.5 - 18.8 \times 5.65 = 66.28$$

The regression line is $\hat{y} = 66.28 + 18.8x$.
Finally we can estimate John's height by plugging in 5.8 into the regression line to get an estimate of $66.28 + 18.8 \times 5.8 = 175.32$.