Homework 9 - *Solution*

MATH 1100-2 - SPRING 2002

1. **12.5.3.** Since $y = 3x^2 + 1$, $dy = y'dx = 6xdx$. Then

$$2ydx - xdy = 2(3x^2 + 1)dx - x \cdot 6xdx = 6x^2dx + 2dx - 6x^2dx = 2dx,$$

as wanted.

2. **12.5.7.** To solve $2ydy = 4xdx$, since the variables are separated we have to integrate both sides:

$$2ydy = 4xdx \Rightarrow \int 2ydy = \int 4xdx \Rightarrow y^2 = 4\frac{x^2}{2} + C = 2x^2 + C.$$

Then, the general solution (in implicit form) is $y^2 = 2x^2 + C$. The explicit form is $y = \pm\sqrt{2x^2 + C}$.

3. **12.5.15.** For $\frac{dy}{dx} = \frac{x^2}{y}$, we start by separating the variables: $ydy = x^2dx$ and then integrate both sides:

$$ydy = x^2dx \Rightarrow \int ydy = \int x^2dx \Rightarrow \frac{y^2}{2} = \frac{x^3}{3} + C,$$

so that the general solution (in implicit form) is $\frac{y^2}{2} = \frac{x^3}{3} + C$. The explicit form of the solution is: $y = \pm\sqrt{\frac{2}{3}x^3 + C'}$.

4. **12.5.29.** For $\frac{dy}{dx} = \frac{x^2}{y^3}$, we start by separating the variables: $y^3dy = x^2dx$ and then integrate both sides:

$$y^3dy = x^2dx \Rightarrow \int y^3dy = \int x^2dx \Rightarrow \frac{y^4}{4} = \frac{x^3}{3} + C,$$

so that the general solution (in implicit form) is $\frac{y^4}{4} = \frac{x^3}{3} + C$. To find the particular solution that has $y = 1$ when $x = 1$ we evaluate at these points and solve for C:

$$\frac{1^4}{4} = \frac{1^3}{3} + C \Rightarrow C = \frac{1}{4} - \frac{1}{3} = -\frac{1}{12}.$$

All together, the particular solution is (in implicit form) $\frac{y^4}{4} = \frac{x^3}{3} - \frac{1}{12}$.

5. **13.1.16.**

$$\sum_{i=3}^{5}(i^2 + 1) = (3^2 + 1) + (4^2 + 1) + (5^2 + 1) = 10 + 17 + 26 = 53.$$

6. **13.1.21.**

$$\sum_{j=1}^{60} 3 = 3\sum_{j=1}^{60} 1 = 3 \cdot 60 = 180.$$

7. **13.1.23.**

$$\sum_{k=1}^{30}(k^2+4k) = \sum_{k=1}^{30} k^2 + \sum_{k=1}^{30} 4k = \frac{30\cdot(30+1)\cdot(2\cdot 30+1)}{6} + 4\sum_{k=1}^{30} k$$
$$= 9455 + 4\frac{30\cdot(30+1)}{2} = 9455 + 1860 = 11315$$

8. **13.2.1.** We have $\int 4x dx = 4\int x dx = 4\frac{x^2}{2} = 2x^2$ (notice that we don't write the constant C because it is not needed for the computation of the definite integral):

$$\int_0^3 4x dx = 2x^2|_0^3 = 2\cdot 3^2 - 2\cdot 0^2 = 2\cdot 9 - 0 = 18.$$

9. **13.2.7.** Since $\int 4\sqrt[3]{x^2}dx = 4\int x^{2/3}dx = 4\cdot\frac{3}{5}\cdot x^{5/3} = \frac{12}{5}x^{5/3}$, we have:

$$\int_0^5 4\sqrt[3]{x^2}dx = \frac{12}{5}x^{5/3}|_0^5 = \frac{12}{5}\cdot 5^{5/3} - \frac{12}{5}\cdot 0^{5/3} = \frac{12}{5}\cdot 5^{5/3} \simeq 35.0882.$$

10. **13.2.13.** Using the substitution $u = x^2+2$, so that $du = 2xdx$ we have $\int(x^2+2)^3xdx = \frac{1}{2}\int u^3du = \frac{u^4}{8} = \frac{1}{8}(x^2+2)^4$. Then:

$$\int_2^4 (x^2+2)^3xdx = \frac{1}{8}(x^2+2)^4|_2^4 = \frac{1}{8}(4^2+2)^4 - \frac{1}{8}(2^2+2)^4 = 13122 - 162 = 12960.$$

11. **13.2.23.** Since $\int \frac{1}{z}dz = \ln(|z|)$, we have

$$\int_1^e \frac{1}{z}dz = \ln(|z|)|_1^e = \ln(|e|) - \ln(|1|) = 1 - 0 = 1.$$

12. **13.2.34.**

    (a) We want the area delimited by $f(x) = \frac{1}{2}x^2 + x + 1$, the x-axis, $x = -2$ and $x = 1$. This area is computed by the definite integral $\int_{-2}^{1}(\frac{1}{2}x^2 + x + 1)dx$.

    (b) We first compute $\int(\frac{1}{2}x^2 + x + 1)dx = \frac{1}{2}\int x^2dx + \int xdx + \int dx = \frac{x^3}{6} + \frac{x^2}{2} + x$. Then:

$$\int_{-2}^{1}(\frac{1}{2}x^2+x+1)dx = (\frac{x^3}{6}+\frac{x^2}{2}+x)|_{-2}^{1} = \frac{1^3}{6}+\frac{1^2}{2}+1-(\frac{(-2)^3}{6}+\frac{(-2)^2}{2}-2)$$
$$= \frac{5}{3} - (-\frac{4}{3}) = 3.$$