Homework 5 - *Solution*

MATH 1100-2 - SPRING 2002

1. **11.1.1.** For $f(x) = 4\ln(x)$:

$$f'(x) = (4\ln(x))' = 4(\ln(x))' = 4 \cdot \frac{1}{x} = \frac{4}{x}.$$

2. **11.1.3.** For $y = \ln(8x)$:

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{\ln(8x)}_{u=8x} = \frac{d\ln(u)}{dx} = \frac{d\ln(u)}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot 8 = \frac{8}{8x} = \frac{1}{x}$$

3. **11.1.5.** For $y = \ln(x^4)$:

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{\ln(x^4)}_{u=x^4} = \frac{d\ln(u)}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{dx^4}{dx} = \frac{1}{x^4} \cdot 4x^3 = \frac{4}{x}$$

A different alternative would be using that $\ln(x^4) = 4\ln(x)$ and then look at exercise **11.1.1**.

4. **11.1.11.** $p = \ln(q^2+1)$, so:

$$\frac{dp}{dq} = \frac{d}{dq}\underbrace{\ln(q^2+1)}_{u=q^2+1} = \frac{d\ln(u)}{dq} = \frac{d\ln(u)}{du} \cdot \frac{du}{dq} = \frac{1}{u} \cdot \frac{d(q^2+1)}{dq} = \frac{1}{q^2+1} \cdot 2q = \frac{2q}{q^2+1}.$$

5. **11.1.21.** $p = \ln(\frac{q^2-1}{q})$. You can do it in two ways: using properties of the logarithm: $p = \ln(q^2-1) - \ln(q)$, so that

$$\frac{dp}{dq} = \frac{d}{dq}(\ln(q^2-1) - \ln(q)) = \frac{d}{dq}(\ln(q^2-1)) - \frac{d}{dq}\ln(q) = \frac{d}{dq}\underbrace{\ln(q^2-1))}_{u=q^2-1} - \frac{1}{q}$$

$$= \frac{d\ln(u)}{dq} - \frac{1}{q} = \frac{d\ln(u)}{du} \cdot \frac{du}{dq} - \frac{1}{q} = \frac{1}{u} \cdot (2q) - \frac{1}{q} = \frac{2q}{q^2-1} - \frac{1}{q}$$

Also,

$$\frac{dp}{dq} = \frac{d}{dq}\underbrace{\ln(\frac{q^2-1}{q})}_{u=\frac{q^2-1}{q}} = \frac{d\ln(u)}{du} \cdot \frac{du}{dq} = \frac{1}{u} \cdot \frac{d}{dq}\frac{q^2-1}{q} = \frac{1}{\frac{q^2-1}{q}} \cdot \frac{2q \cdot q - (q^2-1) \cdot 1}{q^2}$$

$$= \frac{q}{q^2-1} \cdot \frac{q^2+1}{q^2} = \frac{q^2+1}{q(q^2-1)}.$$

A little algebraic massage shows that the two results agree.

6. **11.1.33.** For $y = (\ln((x^4+3)))^2$:

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{(\ln((x^4+3)))^2}_{u=\ln((x^4+3))} = \frac{du^2}{du}\cdot\frac{du}{dx} = 2u\cdot\frac{d}{dx}\underbrace{\ln((x^4+3))}_{v=x^4+3} = 2u\cdot\frac{d\ln(v)}{dx}$$
$$= 2u\cdot\frac{d\ln(v)}{dv}\frac{dv}{dx} = 2u\cdot\frac{1}{v}\cdot\frac{d(x^4+3)}{dx} = 2\ln((x^4+3))\frac{1}{x^4+3}\cdot 4x^3 = 8\frac{\ln((x^4+3))x^3}{x^4+3}.$$

7. **11.2.1.** For $y = 5e^x - x$,

$$y' = (5e^x - x)' = (5e^x)' - (x)' = 5(e^x)' - 1 = 5e^x - 1.$$

8. **11.2.7.** For $y = 6e^{3x^2}$ we have:

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{6e^{3x^2}}_{u=3x^2} = \frac{d6e^u}{dx} = \frac{d6e^u}{du}\cdot\frac{du}{dx} = 6\frac{de^u}{du}\cdot\frac{d(3x^2)}{dx} = 6e^u\cdot 6x = 36xe^{3x^2}.$$

9. **11.2.13.** Given $y = e^{-\frac{1}{x}}$ we compute

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{e^{-\frac{1}{x}}}_{u=-\frac{1}{x}} = \frac{de^u}{dx} = \frac{de^u}{du}\cdot\frac{du}{dx} = e^u\cdot(\frac{1}{x^2}) = \frac{e^{-\frac{1}{x}}}{x^2}.$$

10. **11.2.21.** For $y = \ln(e^{4x}+2)$:

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{\ln(e^{4x}+2)}_{u=e^{4x}+2} = \frac{d\ln(u)}{dx} = \frac{d\ln(u)}{du}\cdot\frac{du}{dx} = \frac{1}{u}\cdot\frac{d}{dx}\underbrace{(e^{4x}+2)}_{v=4x} = \frac{1}{u}\frac{de^v}{dv}\cdot\frac{dv}{dx}$$
$$= \frac{1}{e^{4x}+2}\cdot e^v\cdot 4 = \frac{1}{e^{4x}+2}\cdot e^{4x}\cdot 4$$

11. **11.2.29.** For $y = 6^x$, we remember that $6^x = (e^{\ln(6)})^x = e^{\ln(6)\cdot x}$. Then:

$$\frac{dy}{dx} = \frac{d}{dx}\underbrace{e^{\ln(6)\cdot x}}_{u=\ln(6)x} = \frac{de^u}{dx} = \frac{de^u}{du}\cdot\frac{du}{dx} = e^u\cdot\frac{d(\ln(6)x)}{dx} = e^{\ln(6)x}\cdot\ln(6)\frac{dx}{dx} = \ln(6)\cdot 6^x.$$

12. **11.3.3.** We start from the equation $xy^2 = 8$ and take derivatives on both sides. On the right hand side we have $\frac{d8}{dx} = 0$. On the left hand side:

$$\frac{d}{dx}(xy^2) = \frac{dx}{dx}\cdot y^2 + x\frac{dy^2}{dx} = 1\cdot y^2 + x\frac{dy^2}{dy}\cdot\frac{dy}{dx} = y^2 + x\cdot 2y\cdot\frac{dy}{dx} = y^2 + 2xy\frac{dy}{dx}.$$

Then, equating the (derivatives of the) left and right hand side:

$$0 = y^2 + 2xy\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{y^2}{2xy} = -\frac{y}{2x}.$$

Now we evaluate the derivative at $x = 2$ and $y = 2$:

$$\frac{dy}{dx} = -\frac{2}{2\cdot 2} = -\frac{1}{2}.$$

13. **11.3.17.** From the equation $3x^5 - 5y^3 = 5x^2 + 3y^5$, we compute the derivative of the right hand side:

$$\frac{d}{dx}(5x^2 + 3y^5) = \frac{d5x^2}{dx} + \frac{d3y^5}{dx} = 10x + 3\frac{dy^5}{dy} \cdot \frac{dy}{dx} = 10x + 3 \cdot 5y^4 \cdot \frac{dy}{dx},$$

and the left hand side:

$$\frac{d}{dx}(3x^5 - 5y^3) = \frac{d3x^5}{dx} - \frac{d5y^3}{dx} = 15x^4 - 5\frac{dy^3}{dy} \cdot \frac{dy}{dx} = 15x^4 - 3 \cdot 5y^2 \cdot \frac{dy}{dx}.$$

Now we equate the two derivatives and solve for $\frac{dy}{dx}$:

$$\begin{aligned}
15x^4 - 15y^2 \cdot \frac{dy}{dx} &= 10x + 15y^4 \cdot \frac{dy}{dx} \\
15x^4 - 10x &= 15y^4\frac{dy}{dx} + 15y^2\frac{dy}{dx} \\
15x^4 - 10x &= (15y^4 + 15y^2)\frac{dy}{dx} \\
\frac{15x^4 - 10x}{15y^4 + 15y^2} &= \frac{dy}{dx}.
\end{aligned}$$

Thus:

$$\frac{dy}{dx} = \frac{15x^4 - 10x}{15y^4 + 15y^2} = \frac{3x^4 - 2x}{3y^4 + 3y^2}.$$