Homework 4 - *Solution*

MATH 1100-2 - SPRING 2002

1. **9.7.5.** Using simple properties of addition and multiplication by constants, as well as the derivative of x^n, we have:

$$\begin{aligned} g'(x) &= (5x^3 + \frac{4}{x})' = (5x^3)' + (4 \cdot x^{-1})' = 5(x^3)' + 4(x^{-1})' \\ &= 5 \cdot 3x^2 + 4 \cdot (-1)x^{-2} = 15x^2 - \frac{4}{x^2}. \end{aligned}$$

2. **9.7.7.** For $y = (x^2 - 2)(x + 4)$, using the product rule we have:

$$\begin{aligned} y' &= ((x^2 - 2)(x + 4))' = (x^2 - 2)'(x + 4) + (x^2 - 2)(x + 4)' \\ &= ((x^2)' - (2)')(x + 4) + (x^2 - 2)((x)' + (4)') = (2x - 0)(x + 4) + (x^2 - 2)(1 + 0) \\ &= 2x(x + 4) + x^2 - 2 \end{aligned}$$

3. **9.7.17.** For $y = \frac{(x^2-4)^3}{x^2+1}$, we use the quotient rule first and then the chain rule:

$$\begin{aligned} y' &= (\frac{(x^2 - 4)^3}{x^2 + 1})' = \frac{((x^2 - 4)^3)'(x^2 + 1) - (x^2 - 4)^3(x^2 + 1)'}{(x^2 + 1)^2} \\ &= \frac{3(x^2 - 4)^2 \cdot (x^2 - 4)'(x^2 + 1) - (x^2 - 4)^3((x^2)' + (1)')}{(x^2 + 1)^2} \\ &= \frac{3(x^2 - 4)^2 \cdot ((x^2)' - (4)')(x^2 + 1) - (x^2 - 4)^3(2x + 0)}{(x^2 + 1)^2} \\ &= \frac{3(x^2 - 4)^2 \cdot 2x(x^2 + 1) - (x^2 - 4)^3 2x}{(x^2 + 1)^2} \end{aligned}$$

4. **9.7.33.**

(a)

$$\begin{aligned} \frac{dF_1}{dx} &= \frac{d}{dx}\frac{3\overbrace{(x^4 + 1)}^{u=x^4+1}{}^5}{5} = \frac{d\frac{3}{5}u^5}{dx} = \frac{d\frac{3}{5}u^5}{du} \cdot \frac{du}{dx} = \frac{3}{5} \cdot 5u^4 \cdot \frac{d(x^4 + 1)}{dx} = 3u^4 \cdot 4x^3 \\ &= 12x^3(x^4 + 1)^4 \end{aligned}$$

(b)

$$\begin{aligned} \frac{dF_2}{dx} &= \frac{d}{dx}\frac{3}{5\underbrace{(x^4 + 1)}_{u=x^4+1}{}^5} = \frac{d\frac{3}{5}u^{-5}}{dx} = \frac{d\frac{3}{5}u^{-5}}{du} \cdot \frac{du}{dx} = \frac{3}{5} \cdot (-5)u^{-6} \cdot \frac{d(x^4 + 1)}{dx} \\ &= -3u^{-6} \cdot 4x^3 = -12x^3(x^4 + 1)^{-6} \end{aligned}$$

(c)

$$\frac{dF_3}{dx} = \frac{d}{dx}\frac{\overbrace{(3x^4+1)^5}^{u=3x^4+1}}{5} = \frac{d\frac{1}{5}u^5}{dx} = \frac{d\frac{1}{5}u^5}{du}\cdot\frac{du}{dx} = \frac{1}{5}\cdot 5u^4\cdot\frac{d(3x^4+1)}{dx} = u^4\cdot 3\cdot 4x^3$$
$$= 12x^3(3x^4+1)^4$$

(d)

$$\frac{dF_4}{dx} = \frac{d}{dx}\frac{3}{\underbrace{(5x^4+1)^5}_{u=5x^4+1}} = \frac{d3u^{-5}}{dx} = \frac{d3u^{-5}}{du}\cdot\frac{du}{dx} = 3\cdot(-5)u^{-6}\cdot\frac{d(5x^4+1)}{dx}$$
$$= -3u^{-6}\cdot 5\cdot 4x^3 = -60x^3(5x^4+1)^{-6}$$

5. **9.8.1.** For $f(x) = 4x^3 - 15x^2 + 3x + 2$, we first compute

$$f'(x) = (4x^3 - 15x^2 + 3x + 2)' = (4x^3)' - (15x^2)' + (3x)' + (2)'$$
$$= 4(x^3)' - 15(x^2)' + 3(x)' + 0 = 4\cdot 3x^2 - 15\cdot 2x + 3\cdot 1 = 12x^2 - 30x + 3.$$

Then:

$$f''(x) = (f'(x))' = (12x^2 - 30x + 3)' = (12x^2)' - (30x)' + (3)' = 12(x^2)' - 30(x)' + 0$$
$$= 12\cdot 2x - 30\cdot 1 = 24x - 30$$

6. **9.8.7.** For $y = x^3 - \sqrt{x} = x^3 - x^{\frac{1}{2}}$ we first find

$$y' = (x^3 - x^{\frac{1}{2}})' = (x^3)' - (x^{\frac{1}{2}})' = 3x^2 - \frac{1}{2}x^{-\frac{1}{2}}.$$

Then

$$y'' = (y')' = (3x^2 - \frac{1}{2}x^{-\frac{1}{2}})' = (3x^2)' - (\frac{1}{2}x^{-\frac{1}{2}})' = 3(x^2)' - \frac{1}{2}(x^{-\frac{1}{2}})'$$
$$= 3\cdot 2x - \frac{1}{2}\cdot(-\frac{1}{2})x^{-\frac{3}{2}} = 6x + \frac{1}{4}x^{-\frac{3}{2}}$$

7. **9.8.27.** We have that $f^{(4)}(x) = x(x+1)^{-1} = \frac{x}{x+1}$. Then:

$$f^{(5)}(x) = (f^{(4)}(x))' = (\frac{x}{x+1})' = \frac{(x)'(x+1) - x(x+1)'}{(x+1)^2} = \frac{1\cdot(x+1) - x\cdot 1}{(x+1)^2}$$
$$= \frac{1}{(x+1)^2} = (x+1)^{-2}.$$

And then:

$$f^{(6)}(x) = (f^{(5)}(x))' = ((x+1)^{-2})' = \frac{d}{dx}\underbrace{(x+1)^{-2}}_{u=x+1} = \frac{du^{-2}}{dx} = \frac{du^{-2}}{du}\cdot\frac{du}{dx}$$
$$= (-2)u^{-3}\cdot\frac{d(x+1)}{dx} = -2(x+1)^{-3}\cdot 1 = -2(x+1)^{-3}.$$