Homework 2 - *Solution*

MATH 1100-2 - SPRING 2002

1. **9.3.1.**

   (a) The instantaneous rate of change of $f(x)$ is the derivative $f'(x)$, thus, the instantaneous rate of change at $x = 4$ is the derivative at $x = 4$: $f'(4) = 8 \cdot 4 = 32$.

   (b) The slope of the tangent line at $x = 4$ is, again, the derivative evaluated at $x = 4$, therefore, the slope at that point is $f'(4) = 8 \cdot 4 = 32$.

   (c) This point has y-coordinate $y = f(4) = 64$. In other words, the point is $(4, 64)$.

2. **9.3.3.**

   (a) By definition, we have to find $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h} \\
&= \lim_{h\to 0} \frac{2(x^2 + h^2 + 2xh) - x - h - 2x^2 + x}{h} \\
&= \lim_{h\to 0} \frac{2x^2 + 2h^2 + 4xh - x - h - 2x^2 + x}{h} = \lim_{h\to 0} \frac{2h^2 + 4xh - h}{h} \\
&= \lim_{h\to 0} \frac{h(2h + 4x - 1)}{h} = \lim_{h\to 0}(2h + 4x - 1) = 4x - 1.
\end{aligned}
$$

   Therefore, $f'(x) = 4x - 1$, as required.

   (b) The instantaneous rate of change of $f(x)$ when $x = -1$ is simply $f'(-1)$, which we find evaluating the formula for $f'(x)$. That is, $f'(-1) = 4 \cdot (-1) - 1 = -5$.

   (c) The slope of the tangent to the graph of $f(x)$ at $x = -1$ is, once more, $f'(-1)$, which we have computed before and is -5.

   (d) This point has y-coordinate $y = f(-1) = 2 \cdot (-1)^2 - (-1) = 3$. In other words, the point is $(-1, 3)$.

3. **9.3.11.**

   (a)

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{4(x+h)^2 - 2(x+h) + 1 - (4x^2 - 2x + 1)}{h} \\
&= \lim_{h\to 0} \frac{4x^2 + 4h^2 + 8xh - 2x - 2h + 1 - 4x^2 + 2x - 1}{h} = \lim_{h\to 0} \frac{4h^2 + 8xh - 2h}{h} \\
&= \lim_{h\to 0} \frac{h(4h + 8x - 2)}{h} = \lim_{h\to 0}(4h + 8x - 2) = 8x - 2.
\end{aligned}
$$

   (b) The instantaneous rate of change is the derivative, so we have to find $f'(-3)$. From the previous item we have $f'(3) = 8 \cdot 3 - 2 = 22$.

   (c) The slope of the tangent line at $x = -3$ is, once more, $f'(-3) = 22$.

4. **9.3.19.** Since the derivative is $f'(13) = \lim_{h\to 0} \frac{f(13+h)-f(13)}{h}$, that is, the limiting value of $\frac{f(13+h)-f(13)}{h}$ when h is very small, we can estimate $f'(13)$ by using a small (but not zero) value of h. For example, from the values given in the table we can use $x = 12.99$ which leads to $h = 12.99 - 13 = -0.01$ and

$$f'(13) \simeq \frac{f(12.99) - f(13)}{-0.01} = \frac{17.42 - 17.11}{-0.01} = -31.$$

5. **9.3.25.** Here it is very important to remember that the (infinitesimal) rate of change is the derivative and that the derivative is the slope of the tangent line at a point. Thus, from a graph we can estimate the tangent line, hence its slope, which is the rate of change of f.

   (a) On (a) the tangent lines "point upwards", that is, have positive slope, so that the infinitesimal rate of change is positive. The same holds for (b) and (d).

   (b) On (c) the tangent lines "point downwards", that is, have negative slope, so that the infinitesimal rate of change is negative.

   (c) The rate of change zero means that the tangent line is horizontal, which happens at A, C and E.

6. **9.3.27.**

   (a) The function is continuous at A, B, C and D but not at E, where it is not defined and the graph jumps.

   (b) A function is differentiable at a point if there is a tangent line at the point. Thus the given function is differentiable at A, C and D. It is not differentiable at C because there is no tangent line and is not differentiable at E because the function jumps there (so there is no tangent line).

7. **9.3.35.**

   (a) The average rate of change of D for p going from 1 to 25 is

$$\frac{D(25) - D(1)}{25 - 1} = \frac{\frac{1000}{\sqrt{25}} - 1 - (\frac{1000}{\sqrt{1}} - 1)}{24} = \frac{\frac{1000}{5} - 1 - (\frac{1000}{1} - 1)}{24}$$
$$= \frac{200 - 1 - 1000 + 1}{24} = -\frac{800}{24} \simeq -33.33$$

   (b)

$$\frac{D(100) - D(25)}{100 - 25} = \frac{\frac{1000}{\sqrt{100}} - 1 - (\frac{1000}{\sqrt{25}} - 1)}{75} = \frac{\frac{1000}{10} - 1 - (\frac{1000}{5} - 1)}{75}$$
$$= \frac{100 - 1 - 200 + 1}{75} = -\frac{100}{24} \simeq -1.33$$

8. **9.3.39.**

(a) The marginal revenue is computed by the derivative of R, that is, $R'(x)$. We use the definition:

$$\begin{aligned} R'(x) &= \lim_{h\to 0} \frac{R(x+h)-R(x)}{h} = \lim_{h\to 0} \frac{300(x+h)-(x+h)^2-(300x-x^2)}{h} \\ &= \lim_{h\to 0} \frac{300x+300h-x^2-h^2-2xh-300x+x^2}{h} = \lim_{h\to 0} \frac{300h-h^2-2xh}{h} \\ &= \lim_{h\to 0} \frac{h(300-h-2x)}{h} = \lim_{h\to 0}(300-h-2x) = 300-2x. \end{aligned}$$

(b) It is $R'(50) = 300 - 2\cdot 50 = 200$ and it means that, approximately, the revenue produced by selling an additional unit when 50 units are sold is 200.

(c) It is $R'(200) = 300-2\cdot 200 = -100$ and it means that, approximately, the revenue produced by selling an additional unit when 200 units are sold is -100, that is, you lose 100 for each additional unit!

(d) It is $R'(150) = 300 - 2\cdot 150 = 0$ and it means that, approximately, the revenue produced by selling an additional unit when 150 units are sold is 0.

(e) The marginal revenue passes from being positive to being negative, so it becomes less convenient to sell more than 150 units (because each additional unit sold brings less revenue!).