Homework 1 - *Solution*

MATH 1100-2 - SPRING 2002

1. **9.1.1.** From the graph we see that $f(2) = 1$. We also see that approaching $x = 2$ from either left or right, $f(x)$ approaches 1. Therefore $\lim_{x\to 2} f(x) = 1$.

2. **9.1.7.** From the graph we see that there are two dots on top of $x = -8$, but the convention is that the value of the function is determined by the filled dot. Therefore $f(-8) = -6$.

   Also from the graph, we see that when we approach $x = -8$ from either left or right $f(x)$ approaches 0. Thus, $\lim_{x\to -8} f(x) = 0$. Notice that the value of f at -8 (-6) is irrelevant for this computation.

3. **9.1.11.**

   (a) Approaching $x = -4.5$ from the left we see that $f(x)$ approaches 3, so that $\lim_{x\to -4.5^-} f(x) = 3$.

   (b) Approaching $x = -4.5$ from the right we see that $f(x)$ approaches -6, so that $\lim_{x\to -4.5^+} f(x) = -6$.

   (c) Since the side limits don't agree, $\lim_{x\to -4.5} f(x)$ does not exist.

   (d) From the graph we read $f(-4.5) = -6$.

4. **9.1.13.** The table is shown in Table 1. There we see that as $x \to 2$, $f(x)$ approaches 4. Therefore, $\lim_{x\to 2} f(x) = 4$.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$	4.15789	4.01508	4.0015	3.9985	3.98507	3.85714

Table 1: Exercise 9.1.13.

5. **9.1.17.** Here one must be careful as to what formula to use when computing $f(x)$. The values are shown on Table 2, where we see that $\lim_{x\to 1} f(x)$ does not exist since coming from the left the function approaches 4, whereas coming from the right approaches 5.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	3.5	3.95	3.995	4.996	4.9599	4.59

Table 2: Exercise 9.1.17.

6. **9.1.25.**

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3} x+3 = \lim_{x\to 3} x + \lim_{x\to 3} 3 = 3+3 = 6$$

7. **9.1.31.** Since the function is defined by parts (and we want to compute the limit precisely where the two parts meet) we have to compute the side limits and use the corresponding formula on each side:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} 10-2x = \lim_{x\to 3^-} 10 - \lim_{x\to 3^-} 2x = 10 - (\lim_{x\to 3^-} 2)\cdot(\lim_{x\to 3^-} x) = 10-2\cdot 3 = 4.$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x^2 - x = \lim_{x\to 3^+} x^2 - \lim_{x\to 3^+} x = (\lim_{x\to 3^+} x)^2 - 3 = 3^2 - 3 = 9 - 3 = 6.$$

Thus, since the side limits don't agree, the limit $\lim_{x\to 3} f(x)$ does not exist.

8. **9.1.57.**

$$\lim_{x\to 100} R(x) = \lim_{x\to 100} 1600x - x^2 = (\lim_{x\to 100} 1600)\cdot(\lim_{x\to 100} x) - (\lim_{x\to 100} x)^2 = 1600\cdot 100 - 100^2 = 150000.$$

This value is the revenue when 100 units of the product are sold.

9. **9.1.63.**

(a)

$$\begin{aligned}\lim_{t\to 4} r(t) &= \lim_{t\to 4} \frac{128(t^2+6t)}{(t^2+6t+18)^2} = \frac{\lim_{t\to 4} 128(t^2+6t)}{\lim_{t\to 4}(t^2+6t+18)^2} \\ &= \frac{\lim_{t\to 4} 128\cdot((\lim_{t\to 4} t)^2 + \lim_{t\to 4} 6\cdot \lim_{t\to 4} t)}{((\lim_{t\to 4} t)^2 + \lim_{t\to 4} 6\cdot\lim_{t\to 4} t + \lim_{t\to 4} 18)^2} = \frac{128(4^2+6\cdot 4)}{(4^2+6\cdot 4+18)^2} \\ &= \frac{1280}{841} \simeq 1.52\end{aligned}$$

(b)

$$\begin{aligned}\lim_{x\to 8^-} r(t) &= \lim_{t\to 8^-} \frac{128(t^2+6t)}{(t^2+6t+18)^2} = \frac{\lim_{t\to 8^-} 128(t^2+6t)}{\lim_{t\to 8^-}(t^2+6t+18)^2} \\ &= \frac{\lim_{t\to 8^-} 128\cdot((\lim_{t\to 8^-} t)^2 + \lim_{t\to 8^-} 6\cdot \lim_{t\to 8^-} t)}{((\lim_{t\to 8^-} t)^2 + \lim_{t\to 8^-} 6\cdot\lim_{t\to 8^-} t + \lim_{t\to 8^-} 18)^2} = \frac{128(8^2+6\cdot 8)}{(8^2+6\cdot 8+18)^2} \\ &= \frac{3584}{4225} \simeq 0.85\end{aligned}$$

(c) From the computations we see that the rate of productivity is higher near the lunch break.

10. **9.2.1.**

(a) f is continuous at $x = -5$.

(b) f is not continuous at $x = 1$ because $f(1)$ is not defined. Notice that if we define $f(1) = 2$ then f becomes continuous at 1.

(c) f is not continuous at $x = 3$ because $\lim_{x\to 3} f(x)$ doesn't exist: approaching from the left, the limit is 1, but approaching from the right is -1.

(d) f is not continuous at $x = 0$ because it is not defined (and the side limits are not finite).

11. **9.2.5.** We see that $f(-2) = \frac{0}{-4} = 0$ is defined. Then we compute

$$\lim_{x\to-2} \frac{x^2-4}{x-2} = \frac{\lim_{x\to-2} x^2 - 4}{\lim_{x\to-2} x - 2} = \frac{(\lim_{x\to-2} x)^2 - \lim_{x\to-2} 4}{\lim_{x\to-2} x - \lim_{x\to-2} 2} = \frac{(-2)^2-4}{-2-2} = \frac{0}{4} = 0.$$

So, we see that the limit exists, is finite, and moreover $\lim_{x\to-2} f(x) = f(-2)$. Therefore $f(x)$ is continuous at $x = -2$.

12. **9.2.31.** In order to compute $\lim_{x\to+\infty} \frac{3}{x+1}$ we can see that the denominator grows to infinity, while the numerator remains fixed, so that the limit is 0. Analytically, we can divide both numerator an denominator by x:

$$\lim_{x\to+\infty} \frac{3}{x+1} = \lim_{x\to+\infty} \frac{\frac{3}{x}}{\frac{x+1}{x}} = \lim_{x\to+\infty} \frac{\frac{3}{x}}{\frac{x}{x} + \frac{1}{x}} = \lim_{x\to+\infty} \frac{\frac{3}{x}}{1 + \frac{1}{x}} = \frac{\lim_{x\to+\infty} \frac{3}{x}}{\lim_{x\to+\infty} 1 + \lim_{x\to+\infty} \frac{1}{x}} = \frac{0}{1+0} = 0$$

13. **9.2.35.**

$$\lim_{x\to-\infty} \frac{5x^3-4x}{3x^3-2} = \lim_{x\to-\infty} \frac{\frac{5x^3-4x}{x^3}}{\frac{3x^3-2}{x^3}} = \lim_{x\to-\infty} \frac{\frac{5x^3}{x^3} - \frac{4x}{x^3}}{\frac{3x^3}{x^3} - \frac{2}{x^3}} = \lim_{x\to-\infty} \frac{5 - \frac{4}{x^2}}{3 - \frac{2}{x^3}}$$
$$= \frac{\lim_{x\to-\infty} 5 - \lim_{x\to-\infty} \frac{4}{x^2}}{\lim_{x\to-\infty} 3 - \lim_{x\to-\infty} \frac{2}{x^3}} = \frac{5-0}{3-0} = \frac{5}{3}.$$

14. **9.2.37.**

$$\lim_{x\to+\infty} \frac{3x^2+5x}{6x+1} = \lim_{x\to+\infty} \frac{\frac{3x^2+5x}{x}}{\frac{6x+1}{x}} = \lim_{x\to+\infty} \frac{\frac{3x^2}{x} + \frac{5x}{x}}{\frac{6x}{x} + \frac{1}{x}} = \lim_{x\to+\infty} \frac{3x+5}{6+\frac{1}{x}}$$
$$= \frac{\lim_{x\to+\infty} 3x + \lim_{x\to+\infty} 5}{\lim_{x\to+\infty} 6 + \lim_{x\to+\infty} \frac{1}{x}} = \frac{3(\lim_{x\to+\infty} x) + 5}{6+0} = +\infty$$