3.7 #49-51 Identify the transformation of the graph of \( f(x) = |x| \) and sketch the graph of \( h \). First we write draw the graph of \( f(x) = |x| \) for reference.

\[
\text{#49 } h(x) = |x - 3|
\]

This is the function obtained by putting \((x - 3)\) into absolute values – that is, putting \((x - 3)\) into the function \( f \). So, we have the transformation \( f(x) \to f(x - 3) \), which gives a shift to the right by 3:

\[
\text{#50 } h(x) = |x + 3|
\]

This is the function obtained by putting \((x + 3)\) into absolute values – that is, putting \((x + 3)\) into the function \( f \). So, we have the transformation \( f(x) \to f(x + 3) \), which gives a shift to the left by 3: (turn page)
This is the function we get from subtracting 4 from the expression $|x|$. Note that here the subtraction is outside of the absolute value bars, so the subtraction occurs outside of the function $f$.

Thinking of the function $f$ as a mysterious box we have a diagram

$x \rightarrow f \rightarrow f(x) \rightarrow f(x) - 4$

whereas for #49 and 50 respectively we have diagrams

$x \rightarrow x - 3 \rightarrow f \rightarrow f(x - 3)$

$x \rightarrow x + 3 \rightarrow f \rightarrow f(x + 3)$

For #51, our transformation (of the final result) $f(x) \rightarrow f(x) - 4$ means a shift down of 4 on the graphs:
4.1 #71 Solve the system of equations
\[ \frac{1}{5}x + \frac{1}{2}y = 1 \]
\[ x + y = 20 \]
using the method of substitution.
Step I is to solve for one variable in terms of the other. I’ll solve for \( x \) in the second equation:

\[ x + y = 20 \]
\[ x = 20 - y \]

Step II is to substitute this expression into the other equation:

\[ \frac{1}{5}(20 - y) + \frac{1}{2}y = 1 \]
\[ \frac{1}{5}20 - \frac{1}{5}y + \frac{1}{2}y = 1 \]
\[ 4 - \frac{1}{5}y + \frac{1}{2}y = 1 \]

Use a common denominator to combine the like \( y \) terms. The LCM is 10:

\[ 4 - \frac{20}{10}y + \frac{5}{10}y = 1 \]
\[ 4 + \frac{3}{10}y = 1 \]
\[ \frac{3}{10}y = -3 \]
\[ y = -3 \frac{10}{3} \]
\[ y = -10 \]

Then substitute this into \( x = 20 - y \): \( x = 20 - (-10) = 20 + 10 = 30 \)
The solution is \((30, -10)\)

Alternatively, to avoid all the fraction arithmetic, we could start by multiplying the first equation by 10 on both sides (clearing denominators). We can do this because we know this doesn’t change the solution set of that equation.

Rewriting the first equation \( \frac{1}{5}x + \frac{1}{2}y = 1 \)

\[ 10\left(\frac{1}{5}x + \frac{1}{2}y\right) = 10(1) \]
\[ 10\left(\frac{1}{5}x\right) + 10\left(\frac{1}{2}y\right) = 10 \]
\[ 2x + 5y = 10 \]

So we can rewrite our system equivalently as

\[ 2x + 5y = 10 \]
\[ x + y = 20 \]

Let’s solve this nicer-looking system to see that we get the same solution:

Step I: \( x = 20 - y \)
Step II: \( 2(20 - y) + 5y = 10 \)
\[ 40 - 2y + 5y = 10 \]
\[ 40 + 3y = 10 \]
\[ 3y = -30 \]
\[ y = -10 \]

Resubstitute: \( x = 20 - (-10) = 30 \)
4.1 #95: A farmer wants to mix two types of hay. The first sells for $125 per ton and the second type sells for $75 per ton. The farmer wants a total of 100 tons of hay at a cost of $90 per ton. How many tons of each type of hay should be used in the mixture.

Notice the similarity between this word problem, the mixing saltwater problem, and the grenadine problem – there are two “batches” of something, where each batch has an associated “rate,” and the goal is to create something with a rate somewhere in between.

The unknowns here are the amount from “batch A” (which sells for $125 per ton) and the amount from “batch B” (which sells for $75 per ton). Give these quantities variable names $A$ and $B$ respectively. Let’s model the fact that the total amount of hay is 100 tons in an equation:

\[
A + B = 100
\]

We also know something about the cost per ton. This is a rate associated to the hay. We can use this rate to equate the total costs:

\[
125A + 75B = 9000
\]

So we have a system of two equations, an amount equation and a cost equation:

\[
\begin{align*}
A + B &= 100 \\
125A + 75B &= 9000
\end{align*}
\]

Step I: Solve for one variable in terms of the other – say solve for $A$ in the first equation:

\[
A = 100 - B
\]

Step II: Substitute into the other equation:

\[
\begin{align*}
125(100 - B) + 75B &= 9000 \\
12500 - 125B + 75B &= 9000 \\
12500 - 50B &= 9000 \\
-50B &= -3500 \\
B &= 70
\end{align*}
\]

Resubstitute to get the other value:

\[
A = 100 - B
\]

\[
A = 100 - 70 = 30
\]

Thus, the amount from batch A, the more expensive batch, is 30 tons and the amount from batch B, the less expensive batch, is 70 tons.

Notice that we could also solve this mixing problem by the methods of section 2.3. Check for yourself that under this method we would get the equation $125(100 - B) + 75B = 9000$ from the top of step II, and get the same solution in the end. The method from section 2.3 works by packaging the substitution step into the step of choosing names for the variables.

Now that we know two methods for solving this type of word problem, we should be well-equipped to deal with it if we see it again!