2.2 #49: Solve the proportion \( \frac{y}{6} = \frac{y-2}{4} \):

One good approach to solving proportions is to multiply both sides by the LCM of the denominators (“clear denominators”). The LCM of 4 and 6 is 12, so we multiply both sides by 12 and simplify:

\[
12 \left( \frac{y}{6} \right) = 12 \left( \frac{y-2}{4} \right)
\]

\[
2y = 3(y - 2)
\]

This we can solve using the methods of section 2.1 (see previous solutions):

\[
2y = 3y - 6
\]

\[
-y = -6
\]

\[
y = 6
\]

2.2 #67: A quality control engineer reported that 1.5% of a sample of parts were defective. How big was the sample?

In a word problem, a good first step is to assess what we know and what we aspire to know. We know the number of defective in sample = 3 and % defective in sample = 1.5%. We aspire to know the total in sample, which we can give a variable name “n”. To set this up as a percent problem, the total in sample is the original amount (the number we are taking a percent of), .015 is the percent as a decimal, and the number of defective in sample is the “compared number” (the resulting number after taking a percent). Putting the pieces together gives the equation:

\[
3 = .015n
\]

We divide both sides by .015 to get \( n = 200 \). Thus, the size of the sample was 200.

2.3 #39: Ticket sales for a play total $2200. There are three times as many adult tickets sold as children’s tickets. The prices of the tickets for adults and children are $6 and $4, respectively. Find the number of children’s tickets sold.

Here, we want to know the number of children’s tickets sold. Let’s give this the variable name “n”. Another unknown in the problem here is the number of adult tickets. Rather than giving this a new variable name, we can use the fact that there are three times as many adult tickets sold as children’s tickets to express this number in terms of \( n \): the number of adult tickets is \( 3n \). Now, the total ticket revenue is the sum of adult ticket revenue and children’s ticket revenue. We are told that the total ticket revenue is 2200. We can express adult ticket revenue as the price per adult ticket times the number of adult tickets: \( 6 \cdot 3n \). Similarly, we can express child ticket revenue as the price per child ticket times the number of child tickets: \( 4 \cdot n \). Now we can put the pieces together to get an equation:

\[
2200 = (6 \cdot 3n) + (4 \cdot n)
\]

\[
2200 = 18n + 4n
\]

\[
2200 = 22n
\]

\[
100 = n
\]

Thus, the number of children’s tickets sold was 100.
2.3 #56: Two trucks leave a depot at approximately the same time and travel the same route. How far apart are the trucks after 4 \( \frac{1}{2} \) hours if their average speeds are 52 miles per hour and 56 miles per hour?

For each truck in the problem, we are given a rate and a time. The rate of truck A is 52 (MPH), the rate of truck B is 56, the time for truck A is 4 \( \frac{1}{2} \) = \( \frac{9}{2} \), and the time for truck B is also \( \frac{9}{2} \) (they leave at the same time). What we want to find out is how far apart they are. Since they travel the same route, how far apart they are is the difference of the distances they have traveled: \[ \text{distance of truck B} - \text{distance of truck A} \]

Now we use the fact that \[ \text{distance} = \text{rate} \cdot \text{time} \].

The distance for truck A is \[ 52 \cdot \frac{9}{2} = 234 \]. The distance for truck B is \[ 56 \cdot \frac{9}{2} = 252 \]. Then \[ \text{distance B} - \text{distance A} = 252 - 234 = 18 \]. Thus, the distance between the trucks is 18.

2.3 #63: It takes 30 minutes for a pump to empty a water tank. A larger pump can empty the tank in half the time. How long would it take to empty the tank with both pumps operating?

Here we have a work rate problem: two things are working together at different rates. We first find the work rates: rate for pump A = \( \frac{1}{30} \) (the rate is the reciprocal of the amount of time to complete the task). Pump B finishes the task in 15 minutes, so rate for pump B = \( \frac{1}{15} \). Then we use the model \[ \text{total amount} = \text{rate A} \cdot \text{time} + \text{rate B} \cdot \text{time} \] as in the ice sculpture example in class. We want to find the time to empty 1 tank, so we set the amount completed to 1, give time a variable name “\( t \)” and get the equation

\[ 1 = \frac{1}{30} \cdot t + \frac{1}{15} \cdot t \]

We can clear the denominators in this problem by multiplying through by the LCM of 15 and 30, which is 30:

\[ 30 \cdot 1 = 30(\frac{1}{30} \cdot t + \frac{1}{15} \cdot t) \]

\[ 30 = 30 \frac{1}{30} \cdot t + 30 \frac{1}{15} \cdot t \]

\[ 30 = t + 2t \]

\[ 30 = 3t \]

\[ 10 = t \]

So, it will take the two machines 10 minutes to empty the tank.

2.4 #52: Solve the inequality \( 21x - 11 \leq 6x + 19 \) and sketch the solution on the number line:

The principle of moving the variables to one side and the constants to the other works well for linear inequalities (as it did with linear equations). One way to solve this is by subtracting 6\( x \) from both sides then adding 11 to both sides:

\[ 21x - 11 \leq 6x + 19 \]

\[ 21x - 11 - 6x \leq 19 \]

\[ 15x - 11 \leq 19 \]

\[ 15x \leq 19 + 11 \]

\[ 15x \leq 30 \]

Then we divide both sides by 15. When we do this, we do not have to flip the direction of the inequality, because 15 is positive. Don’t forget that multiplying both sides of an inequality by a negative number causes the inequality to flip.

\[ 15x \leq 30 \]

\[ \frac{1}{15} 15x \leq \frac{1}{15} 30 \]

\[ x \leq 2 \]

So our solution set is the set of \( x \) with \( x \leq 2 \), or the interval \( (-\infty, 2] \). On the number line we include all numbers to the left of and including 2.
2.5: #16: Solve the equation $|s| = 16$:
To solve this, we think about this as a statement about distance on the number line. This says that the distance from $s$ to 0 is 16. The numbers that are distance 16 from 0 are 16 (a distance of 16 to the right) and $-16$ (a distance of 16 to the left). Thus the two solutions are $s = 16$ and $s = -16$.

2.5: #59: Solve the inequality $|x + 6| > 10$:
Translating this into a statement about distance, the distance from $x + 6$ to 0 is greater than 10. The numbers on the number line with distance greater than 10 from 0 are the numbers to the right of 10 and the numbers to the left of $-10$ (draw a picture to convince yourself of this!). So we either need $x + 6$ to be to the right of 10 or $x + 6$ to the left of $-10$. That is, $x + 6 > 10$ or $x + 6 < -10$. Then $x > 4$ or $x < -16$. This gives us the two intervals $(4, \infty)$ and $(-\infty, -16)$. We can write this solution set as $(-\infty, -16) \cup (4, \infty)$ using the union notation.