Quiz #5

Show your work for all problems. Answers without proper justification may not receive full credit. No use of notes, the textbook, or calculators is permitted on the quiz. There are problems on both sides of the page. Don’t forget to write your name on the quiz!

#1. Write each of the following expressions as a single exponential:

a) $e^{x+2}e^3$

$$e^{x+2}e^3 = e^{(x+2)+(3)} = e^{x+5}$$

b) $\frac{2^{3x+1}}{2^{2x}}$

$$\frac{2^{3x+1}}{2^{2x}} = 2^{(3x+1)-(2x)} = 2^x$$

#2. An object tossed upwards at 10 meters per second has height (in meters) at time $t$ given by the function $h(t) = -5t^2 + 10t$. How long will it take the object to reach a height of 4 meters?

We will set the height $h(t)$ equal to 4, and solve for the time $t$.

$$4 = h(t)$$

$$4 = -5t^2 + 10t$$

$$0 = -5t^2 + 10t - 4$$

$$t = \frac{-10 \pm \sqrt{(-10)^2-4(-5)(-4)}}{2(-5)}$$

$$t = \frac{-10 \pm \sqrt{20}}{-10}$$

$$t = 1 \pm \frac{\sqrt{20}}{10}$$

$$t = 1 \pm \frac{\sqrt{5}}{5}$$

We choose the solution $t = 1 - \frac{\sqrt{5}}{5}$ since this is the first time the object achieves the height of 4 meters. The other value $t = 1 + \frac{\sqrt{5}}{5}$ represents when the object has height 4 meters on its way down.
#3. Sketch a graph of the function \( f(x) = -2x^2 + 4x \) on the axes below by using algebra to find the vertex and any \( x- \) and \( y- \) intercepts. Identify these points on the graph.

The vertex has \( x- \) coordinate \( x = \frac{-b}{2a} = \frac{-4}{2(-2)} = 1 \). The \( y- \) coordinate of the vertex is the corresponding function value \( f(1) = -2(1)^2 + 4(1) = 2 \) – the vertex is the point \((1, 2)\).

The \( x- \) intercepts occur where \( y = 0 \):
\[-2x^2 + 4x = 0\]
\[-2x(x - 2) = 0\]
\(x = 0\) or \(x = 2\), so the \( x- \) intercepts are \((0, 0)\) and \((2, 0)\).

The \( y- \) intercepts occur where \( x = 0 \):
\(f(0) = -2(0)^2 + 4(0) = 0\), so the \( y- \) intercept is \((0, 0)\).