Quiz #3

Show your work for all problems. Answers without proper justification may not receive full credit. No use of notes, the textbook, or calculators is permitted on the quiz. There are problems on both sides of the page. Don’t forget to write your name on the quiz!

#1. Simplify \( \frac{(x^3)^2 y^4}{y^{-5}} \)

\[
\begin{align*}
\frac{(x^3)^2 y^4}{y^{-5}} &= \frac{x^6 y^4}{y^{-5}} \\
&= x^6 y^9
\end{align*}
\]

#2. Perform the indicated operations on the given polynomials.

(a) \( (2x^3 - 3x + 7) + (4x - 5) \)

\[
\begin{align*}
(2x^3 - 3x + 7) + (4x - 5) &= 2x^3 - 3x + 7 + 4x - 5 \\
&= 2x^3 + (-3 + 4)x + (7 - 5) \\
&= 2x^3 + x + 2
\end{align*}
\]

(b) \( (2x^3 - 3x + 7) - (4x - 5) \)

\[
\begin{align*}
(2x^3 - 3x + 7) - (4x - 5) &= 2x^3 - 3x + 7 - 4x + 5 \\
&= 2x^3 + (-3 - 4)x + (7 + 5) \\
&= 2x^3 - 7x + 12
\end{align*}
\]

(c) \( (2x^3 - 3x + 7)(4x - 5) \)

\[
\begin{align*}
(2x^3 - 3x + 7)(4x - 5) &= (2x^3 - 3x + 7)(4x) + (2x^3 - 3x + 7)(-5) \\
&= 8x^4 - 12x^2 + 28x - 10x^3 + 15x - 35 \\
&= 8x^4 - 10x^3 - 12x^2 + 43x - 35
\end{align*}
\]
#3. Factor the following polynomials.

(a) \(6x^3 - 9x^2\)
A good technique to try first when factoring is to look for the greatest common monomial factor (GCMF): here it is \(3x^2\).
\[6x^3 - 9x^2 = 3x^2(2x - 3)\]

(b) \(x^3 - 5x^2 + 7x - 35\)
Here the GCMF is 1, so that won't help us factor. However, since the polynomial has 4 terms, grouping is worth a try:
\[\begin{align*}
x^3 - 5x^2 + 7x - 35 &= (x^3 - 5x^2) + (7x - 35) \\
&= x^2(x - 5) + 7(x - 5) \\
&= (x^2 + 7)(x - 5)
\end{align*}\]
Notice that \(x^2(x - 5) + 7(x - 5)\) is not a factorisation of the polynomial since this does not express it as a product of smaller factors (but rather expresses the polynomial as the sum of two things).

(c) \(25x^2 - 36\)
Here the GCMF is 1, so we need a different method to factor this. Note that \(25x^2 = (5x)^2\) and \(36 = 6^2\), so
\[25x^2 - 36 = (5x)^2 - 6^2 = (5x + 6)(5x - 6)\] by the difference of squares formula.

#4. Solve the following system of equations:
\[
\begin{align*}
-2x + y &= -5 \\
3x + 4y &= 2
\end{align*}
\]
We can use either substitution or elimination to solve this system – I’ll give a solution by substitution:
\[-2x + y = -5 \\
y = 2x - 5\]
Substitute into the other equation:
\[
\begin{align*}
3x + 4(2x - 5) &= 2 \\
3x + 8x - 20 &= 2 \\
11x &= 22 \\
x &= 2 \\
y &= 2(2) - 5 = -1
\end{align*}
\]