Exam #2

Show your work for all problems. Answers without proper justification may not receive full credit. No use of notes, the textbook, or calculators is permitted on the exam. There are problems on both sides of each page.

#1. (a) Factor $x^3 + 4x^2 - 9x - 36$ completely.

$$x^3 + 4x^2 - 9x - 36$$

$$= (x^3 + 4x^2) + (-9x - 36)$$

$$= x^2(x + 4) - 9(x + 4)$$

$$= (x^2 - 9)(x + 4)$$

$$= (x + 3)(x - 3)(x + 4)$$

(b) Solve $x^3 + 4x^2 - 9x - 36 = 0$.

$$x^3 + 4x^2 - 9x - 36 = 0$$

$$(x + 3)(x - 3)(x + 4) = 0$$

$$x + 3 = 0, x - 3 = 0, \text{ or } x + 4 = 0$$

$$x = -4, -3, 3$$

#2. Simplify the following expressions.

(a) $\frac{(x^2y)^3}{x^2y^3}$

$$= \frac{x^6y^3}{x^2y^3}$$

$$= x^4y^0$$

$$= x^4$$

(b) $\sqrt[3]{40x^3y^4}$

$$\sqrt[3]{40x^3y^4}$$

$$= \sqrt[3]{40} \sqrt[3]{x^3} \sqrt[3]{y^4}$$

$$= 2x y \sqrt[3]{5y}$$

#3. Use polynomial long division to find $2x^2 - 3x - 7 \div x + 2$.

$$\frac{2x - 7}{x + 2}$$

$$= 2x^2 - 3x - 7$$

$$- 2x^2 - 4x$$

$$- 7x - 7$$

$$7x + 14$$

$$7$$
#4. Perform the indicated operations and write the following in the standard $a+bi$ form:
    (a) $(3 - 5i)(2 + i)$

    $(3 - 5i)(2 + i) = 6 + 3i - 10i - 5i^2 = 6 - 7i + 5 = 11 - 7i$

    (b) $\frac{3 - 5i}{2 + i}$

    $\frac{3 - 5i}{2 + i} = \frac{(3 - 5i)(2 - i)}{(2 + i)(2 - i)} = \frac{1 - 13i}{5}$

#5. Solve for $x$ where $\frac{4}{x} = \frac{x + 1}{5}$.

    $5 \cdot \frac{4}{x} = 5x \cdot \frac{x + 1}{5}$
    $20 = x(x + 1)$
    $0 = x^2 + x - 20$
    $0 = (x + 5)(x - 4)$
    $x = -5, 4$

#6. Solve for $v$ where $3 + \sqrt{2v - 1} = 10$.

    $3 + \sqrt{2v - 1} = 10$
    $\sqrt{2v - 1} = 7$
    $(\sqrt{2v - 1})^2 = 7^2$
    $2v - 1 = 49$
    $2v = 50$
    $v = 25$
    check: $3 + \sqrt{2(25) - 1} = 10$

#7. Use the method of completing the square to solve for $y$ where $2y^2 - 4y - 4 = 0$.

    $2y^2 - 4y - 4 = 0$
    $y^2 - 2y - 2 = 0$
    $y^2 - 2y + 1 = 3$
    $(y - 1)^2 = 3$
    $y - 1 = \pm \sqrt{3}$
    $y = 1 \pm \sqrt{3}$

#8. Perform the indicated operations:
    (a) $\frac{2x + 1}{x - 4} - \frac{3}{x}$

    $\frac{2x + 1}{x - 4} - \frac{3}{x} = \frac{(2x + 1)(x) - 3(x - 4)}{x(x - 4)} = \frac{2x^2 + x - 3x + 12}{x(x - 4)} = \frac{2x^2 - 2x + 12}{x(x - 4)}$
\[(\sqrt{5} - 2)(4 + 2\sqrt{5})
\]

\[(\sqrt{5} - 2)(4 + 2\sqrt{5})
\]

\[= 4\sqrt{5} + 2\sqrt{5}\sqrt{5} - 8 - 4\sqrt{5}
\]

\[= 2(5) - 8
\]

\[= 2
\]

(bonus) If Lajos and Mladen can paint a statue together in 6 minutes, and Lajos by himself takes 5 minutes longer to paint the statue than Mladen does by himself, how long does it take Mladen to paint the statue?

Put \(M\) for Mladen’s time. Lajos’ time is \(M + 5\). Malden’s work rate is \(\frac{1}{M}\), Lajos’ is \(\frac{1}{M+5}\), and together \(\frac{1}{6}\). Work rates add:

\[
\frac{1}{M} + \frac{1}{M+5} = \frac{1}{6}
\]

\[
6(M+5)\frac{1}{M} + 6M(M+5)\frac{1}{M+5} = 6M(M+5)\frac{1}{6}
\]

\[
6(M+5) + 6M = M(M+5)
\]

\[
6M + 30 + 6M = M^2 + 5M
\]

\[
0 = M^2 - 7M - 30
\]

\[
0 = (M + 3)(M - 10)
\]

\[
M = 10, -3
\]

The solution \(M = -3\) doesn’t make sense in the context of the problem, so it will take Mladen 10 minutes to paint the statue.