| Copyright \& License <br> Copyright (c) 2007 Jason Underdown Some rights reserved. <br> Electrodynamics | Definition <br> gradient <br> Electrodynamics |
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| Definition <br> the vector operator $\nabla$ <br> Electrodynamics | Definition <br> divergence <br> Electrodynamics |
| Definition curl <br> Electrodynamics | Definition <br> 5 species of second derivatives <br> Electrodynamics |
| Theorem <br> curl-less or irrotational fields <br> Electrodynamics | Theorem <br> divergence-less or solenoidal fields <br> Electrodynamics |
| Theorem <br> gradient theorem <br> Electrodynamics | Theorem <br> Green's theorem <br> Electrodynamics |

The gradient $\nabla T$ points in the direction of maximum increase of the function $T$.

$$
\nabla T \equiv \hat{\mathbf{x}} \frac{\partial T}{\partial x}+\hat{\mathbf{y}} \frac{\partial T}{\partial y}+\hat{\mathbf{z}} \frac{\partial T}{\partial z}
$$

The magnitude $|\nabla T|$ is the slope along this direction.

$$
\begin{aligned}
\nabla \cdot \mathbf{v} & =\left(\hat{\mathbf{x}} \frac{\partial}{\partial x}+\hat{\mathbf{y}} \frac{\partial}{\partial y}+\hat{\mathbf{z}} \frac{\partial}{\partial z}\right) \cdot\left(v_{x} \hat{\mathbf{x}}+v_{y} \hat{\mathbf{y}}+v_{z} \hat{\mathbf{z}}\right) \\
& =\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{y}}{\partial y}+\frac{\partial v_{z}}{\partial z}
\end{aligned}
$$

The divergence is a measure of how much the vector function $\mathbf{v}$ spreads out from the point in question.

By applying $\nabla$ twice we can construct five species of second derivatives.

1. divergence of a gradient $\nabla \cdot(\nabla T)=\nabla^{2}$ (Laplacian)
2. curl of a gradient $\nabla \times(\nabla T)=0$ (always)
3. gradient of a divergence $\nabla(\nabla \cdot \mathbf{v})$ (seldom occurs)
4. divergence of a curl $\nabla \cdot(\nabla \times \mathbf{v})=0$ (always)
5. curl of a curl $\nabla \times(\nabla \times \mathbf{v})=\nabla(\nabla \cdot \mathbf{v})-\nabla^{2} \mathbf{v}$
By applying $\nabla$ twice we can construct five species of second
derivatives.
6. divergence of a gradient $\nabla \cdot(\nabla T)=\nabla^{2}$ (Laplacian)
7. curl of a gradient $\nabla \times(\nabla T)=0$ (always)
8. gradient of a divergence $\nabla(\nabla \cdot \mathbf{v})$ (seldom occurs)
9. divergence of a curl $\nabla \cdot(\nabla \times \mathbf{v})=0$ (always)
10. curl of a curl $\nabla \times(\nabla \times \mathbf{v})=\nabla(\nabla \cdot \mathbf{v})-\nabla^{2} \mathbf{v}$

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$$
\nabla \equiv \hat{\mathbf{x}} \frac{\partial}{\partial x}+\hat{\mathbf{y}} \frac{\partial}{\partial y}+\hat{\mathbf{z}} \frac{\partial}{\partial z}
$$

For a given vector field $\mathbf{F}$ the following statements are equivalent, i.e. each implies the others.

1. $\nabla \cdot \mathbf{F}=0$ everywhere
2. $\int \mathbf{F} \cdot d \mathbf{a}$ is independent of surface

$$
\nabla \times \mathbf{v}=\left|\begin{array}{ccc}
\hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
v_{x} & v_{y} & v_{z}
\end{array}\right|
$$

The curl is a measure of how much the vector field "curls around" the point in question.
3. $\oint \mathbf{F} \cdot d \mathbf{a}=0$ over any closed surface
4. $\mathbf{F}=\nabla \times \mathbf{A}$ for some vector potential $\mathbf{A}$

For a given vector field $\mathbf{F}$ the following statements are equivalent, i.e. each implies the others.

1. $\nabla \times \mathbf{F}=0$ everywhere
2. $\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{F} \cdot d \mathbf{l}$ is path independent
3. $\oint \mathbf{F} \cdot d \mathbf{l}=0$ on any closed loop
4. $\mathbf{F}=-\nabla V$ for some scalar potential $V$

$$
\int(\nabla \cdot \mathbf{A}) d V=\oint \mathbf{A} \cdot d \mathbf{a}
$$

$$
\int_{\mathbf{a}}^{\mathbf{b}}(\nabla f) \cdot d \mathbf{l}=f(\mathbf{b})-f(\mathbf{a})
$$

| THEOREM |  |  |
| :--- | :--- | :--- |
| Stokes' theorem |  |  |
| ELECTRODYNAMICS |  |  |


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