Thou Shalt Not Distribute Powers or Radicals

Thou Shalt Not Split a Denominator

Thou Shalt Not Cancel Terms in a Fraction

1.1 Definition

algebraic expression
terms & factors

1.1 Definition

equation

1.1 Definition

linear equation

1.1 Definition

identity

polynomial
Thou shalt not distribute powers or radicals.

\[(a \pm b)^n \neq a^n \pm b^n\]
\[\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}\]
\[\sqrt[n]{a^n \pm b^n} \neq a \pm b\]

Lest I smite thee with a failing grade!

Thou shalt not cancel terms in a fraction. Only factors may be cancelled, thus thou must factor first!

\[\frac{a + b}{a + c} \neq \frac{1 + b}{1 + c}\]
\[\frac{a + b}{ac} \neq \frac{1 + b}{c}\]

This is an algebraic abomination.

 Thou shalt not split a denominator.
(This is distributing a -1 power in disguise.)

\[\frac{1}{a + b} \neq \frac{1}{a} + \frac{1}{b}\]

Do not succumb to such temptation.

An expression obtained by performing additions, subtractions, multiplications, divisions, powers or extractions of roots with one or more real numbers or variables is called an **algebraic expression**.

Think of it as a fragment of a complete mathematical statement, and as such, it can only be simplified but not solved for the variable(s).

A **term** is any algebraic expression participating in addition or subtraction.

A **factor** is any algebraic expression participating in multiplication or division.

An **identity** is a special type of equation which is always true for all values of its variables. It tells you how to rewrite an expression in a different but equivalent way.

For example, the “difference of two squares” formula is an identity.

\[x^2 - a^2 = (x + a)(x - a)\]

A **polynomial** is a special algebraic expression of the form:

\[a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0\]

Where each \(a_i\) is a real number. For example

\[x^3 - 3x^2 + 3x - 1\]

is a polynomial.

An **equation** is a mathematical statement that equates two algebraic expressions. An equation **must** have an equal sign.

We typically solve equations for a particular variable. For example, \(3x - 4 = 2\).

A **linear equation** is an equation that can be written in the form

\[ax + b = c\]

where \(a \neq 0\) and \(a, b, c \in \mathbb{R}\).
1.1 Definition

\textit{rational equation}

\textit{domain & range}

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1.1 Algorithm

\textit{determining the domain}

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1.2 Definition

\textit{linear inequality}

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1.2 Definition

\textit{caveat when solving linear inequalities}

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1.3 Definition

\textit{linear equation in two variables}

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1.3 Definition

\textit{input variable}

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1.3 Definition

\textit{output variable}

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1.3 Definition

\textit{y–intercept}

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1.3 Definition

\textit{x–intercept}

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The **domain** of a function is the set of allowable inputs.

The **range** of a function is the set of values which correspond to at least one value in the domain.

An **rational equation** is an equation that contains rational expressions, which are fractions which have polynomials in the numerator and/or denominator.

When solving a rational equation it is important to determine the domain, and exclude any values from the solution set which are not in the domain.

A **linear inequality** is an inequality that can be written in the form

\[ ax + b \leq c \]

where \( a \neq 0 \) and \( a, b, c \in \mathbb{R} \).

The inequality symbol may be any one of \(<, >, \leq, \geq\).

If the domain is unspecified, then it includes all real numbers (\( \mathbb{R} \)) except

1. Values that make the denominator 0, and
2. Values that result in an even root of a negative number, e.g. \( \sqrt{-5} \) or \( \sqrt{-2} \).

We solve linear inequalities in the same manner as solving linear equations. We can

1. Add (or subtract) any quantity to both sides of the inequality, and
2. Multiply (or divide) both sides of the inequality by any nonzero value.

*However, when multiplying by a negative number, we must flip the inequality.*

A **linear equation in two variables** is any equation that can be written in the form

\[ y = ax + b \]

where \( a, b \in \mathbb{R} \).

The **output variable** or **dependent variable** is the variable that is graphed on the vertical axis in a Cartesian coordinate system. It is often denoted by \( y \).

The **input variable** or **independent variable** is the variable that is graphed on the horizontal axis in a Cartesian coordinate system. It is often denoted by \( x \).

The point on a line that crosses the \( x \)-axis. To find the \( x \)-intercept, set the equation of a line equal to zero and solve for \( x \), i.e. solve

\[ mx + b = 0 \]

The point on a line that crosses the \( y \)-axis. When an equation of a line is written

\[ y = mx + b \]

it is the \( b \) value.
1.3 Definition

**slope of a line**

**parallel lines**

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1.3 Definition

**perpendicular lines**

**slope–intercept form**

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1.4 Definition

**point–slope form**

**system of equations**

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1.5 Definition

**solution set**

**revenue & cost**

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1.5 Definition

**profit**

**break–even point**

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Two lines are parallel if and only if they have the same slope.

The equation of a line is in slope–intercept form if it looks like
\[ y = mx + b \]
Where \( m \), the slope of the line, and \( b \), the \( y \)-intercept of the line are constants.

A system of equations is a set of two or more equations in two or more unknowns (variables).

Revenue is the amount of money earned in a business, usually denoted \( R(x) \).

Cost is the amount of money spent to produce and sell a product or service, usually denoted \( C(x) \). There are two types: fixed and variable.

The point where costs and revenue are equal. Also the point where \( P(x) = 0 \).

The slope of a line is found from two points, \((x_1, y_1)\) and \((x_2, y_2)\)
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
Remember it as “rise over run.”

If a line has slope \( m \), then any line that is perpendicular will have slope
\[ m_\perp = -\frac{1}{m} \]
Where the symbol \( m_\perp \) is read “\( m \) perp.” Remember the perpendicular slope is “the negative reciprocal.”

The equation of a line is in point–slope form if it looks like
\[ y - y_1 = m(x - x_1) \]
where \( m \) is the slope of the line, and \((x_1, y_1)\) is a point on the line.

The solution set of a system is the set of all points that lie on all lines in the system. That is, the set of all pairs \((x, y)\) that satisfy all equations simultaneously. The solution set may:
1. be empty (no solution—parallel lines)
2. contain one point (unique solution—crossing lines)
3. contain an infinite number of points (infinitely many solutions—colinear lines)

Profit is revenue minus costs.
\[ P(x) = R(x) - C(x) \]
1.5 Definition

*marginal*

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1.5 Definition

*supply equation*

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1.5 Definition

*demand equation*

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1.5 Definition

*equilibrium point*

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1.6 Definition

*linear inequality in two variables*

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Definition

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Definition

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Definition

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The **supply equation** relates $p$ and $q$ and explains how willing producers are to supply product at various prices.

The point where supply and demand are equal.

The **demand equation** relates $p$ and $q$ and explains how much of a product consumers will buy at various prices.

**Marginal** is an adjective that describes how much something changes for one more unit sold.

For example, we could ask for marginal profit, which would indicate how much profit changed for one more unit sold. Marginal profit is the slope of the profit function.

An inequality that can be written in the form

$$ax + by \leq c$$

where $a, b, c \in \mathbb{R}$ and the inequality can be any of $<, >, \leq, \geq$. 