1. There are three elementary symmetric polynomials in three unknowns:

$$\begin{aligned}\sigma_1 &= x_1 + x_2 + x_3 \\ \sigma_2 &= x_1x_2 + x_1x_3 + x_2x_3 \\ \sigma_3 &= x_1x_2x_3\end{aligned}$$

The following calculation shows how to express the symmetric polynomial $x_1^2+x_2^2+x_3^2$ in terms of the elementary symmetric polynomials above.

$$\begin{aligned}\sigma_1^2 &= (x_1 + x_2 + x_3)^2 \\ &= (x_1 + x_2 + x_3)(x_1 + x_2 + x_3) \\ &= x_1^2 + x_2^2 + x_3^2 + 2(x_1x_2 + x_1x_3 + x_2x_3) \\ &= x_1^2 + x_2^2 + x_3^2 + 2\sigma_2\end{aligned}$$

Thus $x_1^2 + x_2^2 + x_3^2 = \sigma_1^2 - 2\sigma_2$.

Express the symmetric polynomial $x_1^3 + x_2^3 + x_3^3$ in terms of the elementary symmetric polynomials σ_1, σ_2 and σ_3. Hint: start with σ_1^3, and consider various products of σs.

Solution: $x_1^3 + x_2^3 + x_3^3 = \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3$