

1. Omar Khayyam developed a technique which used the point of intersection of two conic sections in the plane to geometrically estimate solutions to cubic polynomial equations. In our class discussion, I mentioned that the following equation is actually the equation of a circle in disguise:

$$\frac{q-x}{y} = \frac{y}{x}.$$

Show that this equation is equivalent to the equation of a circle with center located at  $(q/2, 0)$ .

**Solution:** Your answer here...

2. Solve the following cubic equation in  $y$  via Tartaglia's method.

$$y^3 - 3y^2 - y + 3 = 0$$

First, *reduce* it to a cubic equation in  $x$  with no quadratic term, i.e. an equation of the form  $x^3 + px + q = 0$ .

(Hint 1: Set  $y = x + k$ , and then choose  $k$  such that the coefficient of the quadratic term is 0.)

Once you know  $p$  and  $q$  use Cardano's formula below to solve for  $x$  and finally use your values for  $x$  and  $k$  to solve for the root  $y = x + k$ .

(Hint 2: Cardano's formula will give you an expression involving complex numbers, but this expression simplifies to an integer.)

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} \quad (\text{Cardano's Formula})$$

Show your work to receive full credit.

**Solution:** Your answer here...