In each problem below use the Euclidean Algorithm and show your steps, you may omit division calculations. Do not find the gcd by finding the prime factorization of each number. (The `\begin{align*}...\end{align*}` environment and the `\gcd` command will be useful for this assignment.)

1. Find the gcd of 72 and 56, and write an integral linear combination of 72 and 56 which equals their gcd. That is, find s and t such that $72 \cdot s + 56 \cdot t = \gcd(72, 56)$.

Solution:

$$72 = 56 \cdot 1 + 16 \quad \longrightarrow \quad 16 = 72 - 56 \cdot 1 \tag{1a}$$
$$56 = 16 \cdot 3 + 8 \quad \longrightarrow \quad 8 = 56 - 16 \cdot 3 \tag{1b}$$
$$16 = 8 \cdot 2 + 0$$

$$\begin{aligned} 8 &= 56 - 16 \cdot 3 \\ 8 &= 56 - (72 - 56 \cdot 1) \cdot 3 \\ 8 &= 56 \cdot 4 - 72 \cdot 3 \end{aligned}$$

Hence $56(4) + 72(-3) = 8 = \gcd(72, 56)$.

2. Find the gcd of 99 and 63, and write an integral linear combination of 99 and 63 which equals their gcd. That is, find s and t such that $99 \cdot s + 63 \cdot t = \gcd(99, 63)$.

Solution:

$$99 = 63 \cdot 1 + 36 \quad \longrightarrow \quad 36 = 99 - 63 \cdot 1 \tag{2a}$$
$$63 = 36 \cdot 1 + 27 \quad \longrightarrow \quad 27 = 63 - 36 \cdot 1 \tag{2b}$$
$$36 = 27 \cdot 1 + 9 \quad \longrightarrow \quad 9 = 36 - 27 \cdot 1 \tag{2c}$$
$$27 = 9 \cdot 3 + 0$$

$$\begin{aligned} 9 &= 36 - 27 \cdot 1 \\ 9 &= 36 - (63 - 36 \cdot 1) \cdot 1 \\ 9 &= 36 \cdot 2 - 63 \cdot 1 \\ 9 &= (99 - 63 \cdot 1) \cdot 2 - 63 \cdot 1 \\ 9 &= 99 \cdot 2 - 63 \cdot 3 \end{aligned}$$

Hence $99(2) + 63(-3) = 9 = \gcd(99, 63)$.

3. Find the gcd of 423 and 128, and write an integral linear combination of 423 and 128 which equals their gcd. That is, find s and t such that $423 \cdot s + 128 \cdot t = \gcd(423, 128)$.

Solution:

$$
\begin{aligned}
423 &= 128 \cdot 3 + 39 & \longrightarrow && 39 &= 423 - 128 \cdot 3 && \text{(3a)}\\
128 &= 39 \cdot 3 + 11 & \longrightarrow && 11 &= 128 - 39 \cdot 3 && \text{(3b)}\\
39 &= 11 \cdot 3 + 6 & \longrightarrow && 6 &= 39 - 11 \cdot 3 && \text{(3c)}\\
11 &= 6 \cdot 1 + 5 & \longrightarrow && 5 &= 11 - 6 \cdot 1 && \text{(3d)}\\
6 &= 5 \cdot 1 + 1 & \longrightarrow && 1 &= 6 - 5 \cdot 1 && \text{(3e)}\\
5 &= 1 \cdot 5 + 0 & & & & & &
\end{aligned}
$$

$$
\begin{aligned}
1 &= 6 - 5 \cdot 1\\
1 &= 6 - (11 - 6 \cdot 1) \cdot 1\\
1 &= 6 \cdot 2 - 11 \cdot 1\\
1 &= (39 - 11 \cdot 3) \cdot 2 - 11 \cdot 1\\
1 &= 39 \cdot 2 - 11 \cdot 7\\
1 &= 39 \cdot 2 - (128 - 39 \cdot 3) \cdot 7\\
1 &= 39 \cdot 23 - 128 \cdot 7\\
1 &= (423 - 128 \cdot 3) \cdot 23 - 128 \cdot 7\\
1 &= 423 \cdot 23 - 128 \cdot 76
\end{aligned}
$$

Hence $423(23) + 128(-76) = 1 = \gcd(423, 128)$.