1. Prove that the kernel of a homomorphism is a subgroup of the domain of the homomorphism.

   Hints: Since the domain is a group, and since any subgroup shares the same operation as its parent there is no need to show that the operation is associative. Let $\varphi : G \to G'$ be a homomorphism, then you must show three things:

   1. **closure:** Pick two arbitrary elements say $a, b \in \ker(\varphi)$ and show that their product must necessarily also be an element of $\ker(\varphi)$, i.e. show $\varphi(ab) = 1$.
   2. **identity:** Show that $1 \in \ker(\varphi)$.
   3. **inverses:** Show that if $a \in \ker(\varphi)$, then $a^{-1} \in \ker(\varphi)$.

   Solution: Your answer here...

2. Let G be a group, the map $f : G \to G$ given by $f : g \mapsto g^{-1}$ is not always a homomorphism. Why not? What property must G have to make it a homomorphism?

   Hint: Consider the product of two elements in G say ab, then $f(ab) = [ab]^{-1}$, but you can actually figure out how to write $[ab]^{-1}$ in terms of a^{-1} and b^{-1} if you use the definition of inverses (found in the group definition).

   Solution: Your answer here...

3. Let G be a group, prove that "the conjugation by g" map $\varphi_g : G \to G$ given by $\varphi_g : a \mapsto gag^{-1}$ is a homomorphism.

   Hint: You want to show that given $a, b \in G, \varphi_g(ab) = \varphi_g(a)\varphi_g(b)$. This can be done by inserting a special form of the identity element, namely, $g^{-1}g$, into the image of ab.

   Solution: Your answer here...