1. Determine which pairs of symmetries generate the entire set of symmetries of the equilateral triangle, i.e. $\{R_0, R_{120}, R_{240}, F_1, F_2, F_3\}$. Show how to generate each symmetry using just elements in the pair. Can this set be generated by a single symmetry?

Solution: There are nine possible pairs which generate D_6. The tables below show how to generate all six elements of the group using just elements in the pair above it. The compositions given below are not unique, that is there are an infinite number of ways to generate an element given the elements in the pair, so there are many correct answers.

There is no single element which generates the entire group.

(R_{120}, F_1) :

$R_0 = R_{120}^3$	$F_1 = F_1$
$R_{120} = R_{120}$	$F_2 = R_{120}^2 \circ F_1$
$R_{240} = R_{120}^2$	$F_3 = R_{120} \circ F_1$

(R_{240}, F_1) :

$R_0 = R_{240}^3$	$F_1 = F_1$
$R_{120} = R_{240}^2$	$F_2 = R_{240} \circ F_1$
$R_{240} = R_{240}$	$F_3 = F_1 \circ R_{240}$

(R_{120}, F_2) :

$R_0 = R_{120}^3$	$F_1 = R_{120} \circ F_2$
$R_{120} = R_{120}$	$F_2 = F_2$
$R_{240} = R_{120}^2$	$F_3 = F_2 \circ R_{120}$

(R_{240}, F_2) :

$R_0 = R_{240}^3$	$F_1 = F_2 \circ R_{240}$
$R_{120} = R_{240}^2$	$F_2 = F_2$
$R_{240} = R_{240}$	$F_3 = R_{240} \circ F_2$

(R_{120}, F_3) :

$R_0 = R_{120}^3$	$F_1 = F_3 \circ R_{120}$
$R_{120} = R_{120}$	$F_2 = R_{120} \circ F_3$
$R_{240} = R_{120}^2$	$F_3 = F_3$

(R_{240}, F_3) :

$R_0 = R_{240}^3$	$F_1 = R_{240} \circ F_3$
$R_{120} = R_{240}^2$	$F_2 = F_3 \circ R_{240}$
$R_{240} = R_{240}$	$F_3 = F_3$

(F_1, F_2) :

$R_0 = F_1^2$	$F_1 = F_1$
$R_{120} = F_1 \circ F_2$	$F_2 = F_2$
$R_{240} = F_2 \circ F_1$	$F_3 = F_1 \circ F_2 \circ F_1 = F_2 \circ F_1 \circ F_2$

(F_2, F_3) :

$R_0 = F_2^2$	$F_1 = F_2 \circ F_3 \circ F_2 = F_3 \circ F_2 \circ F_3$
$R_{120} = F_2 \circ F_3$	$F_2 = F_2$
$R_{240} = F_3 \circ F_2$	$F_3 = F_3$

(F_1, F_3) :

$R_0 = F_1^2$	$F_1 = F_1$
$R_{120} = F_3 \circ F_1$	$F_2 = F_1 \circ F_3 \circ F_1 = F_3 \circ F_1 \circ F_3$
$R_{240} = F_1 \circ F_3$	$F_3 = F_3$

2. Create a Cayley table for the symmetries of a square.
Hints: This group has eight symmetries. There are four rotational symmetries and four flip symmetries. Two of the flips are with respect to axes through diagonal vertices and two flips are with respect to axes through opposite midpoints i.e. one horizontal axis and one vertical axis.

Use similar notation as in the triangle case, let $r = R_{90}$ and let f be a flip through the vertical axis. In other words, your symmetries should be named only using the characters r and f with exponents only on r. Furthermore, any symmetry names with an f in them should have it at the end.

Solution:

$\circ$	1	r	r^2	r^3	f	rf	r^2f	r^3f
1	1	r	r^2	r^3	f	rf	r^2f	r^3f
r	r	r^2	r^3	1	rf	r^2f	r^3f	f
r^2	r^2	r^3	1	r	r^2f	r^3f	f	rf
r^3	r^3	1	r	r^2	r^3f	f	rf	r^2f
f	f	r^3f	r^2f	rf	1	r^3	r^2	r
rf	rf	f	r^3f	r^2f	r	1	r^3	r^2
r^2f	r^2f	rf	f	r^3f	r^2	r	1	r^3
r^3f	r^3f	r^2f	rf	f	r^3	r^2	r	1