Name ________________________________

Typesetting with LaTeX

1. [5 pts.] Choose the correct list of special characters in LaTeX. Just write the letter corresponding to the list in the blank below.

    A. `# $ % ^ & _ { } ~ \`

    B. `$ % ^ & * _ { } ~ \`

    C. `# $ % ^ & _ { } * \`

    D. `# $ % ^ & _ { } ~ +`

1. ____**A**____

2. [10 pts.] Write the LaTeX code which generates the following table in the box below:

number	English	Spanish
1	one	uno
2	two	dos

Solution:

```
\begin{tabular}{cll}
  number & English & Spanish \\
  \hline
  1 & one & uno \\
  2 & two & dos \\
\end{tabular}
```

3. [10 pts.] Write the LaTeX code which generates the following equation in the box below. You may assume that the `amsmath` package has been loaded.

$$\sqrt[3]{2-\sqrt{3}} = \dfrac{1}{\sqrt[3]{2+\sqrt{3}}} \tag{1}$$

Solution:

```
\begin{equation}
  \sqrt[3]{2 - \sqrt{3}} = \dfrac{1}{\sqrt[3]{2 + \sqrt{3}}}
\end{equation}
```

Symmetries and Groups of Symmetries

For the next few questions, let $\{R_0, R_{120}, R_{240}, F_1, F_2, F_3\}$ be the set of symmetries of the equilateral triangle.

4. [5 pts.] $R_{240} \circ F_2 \circ R_{120} = \underline{\quad}$?

4. F_1

5. [5 pts.] Solve the following equation for X:

$$R_{240} \circ X \circ F_3 = R_0$$

5. F_2

6. [5 pts.] How can you generate F_1 via the two generators F_2 and F_3? In other words, what string of flips consisting only of F_2 and F_3 is equivalent to F_1? There is more than one correct answer, and short answers are appreciated.

6. $F_2 \circ F_3 \circ F_2$

7. [5 pts.] Write the following permuatation given in tabular notation in cycle notation. That is, write it as a product of disjoint cycles.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 7 & 4 & 1 & 6 & 2 & 5 \end{pmatrix}$$

7. (1 3 4) (2 7 5 6)

8. Multiply (compose) the following permutations written in cycle notation.
   (a) [5 pts.] (3 6 8) (2 6 8 3) (2 3)

   (a) (3 8 6)

   (b) [5 pts.] (1 2 3) (4 5 6)

   (b) (1 2 3) (4 5 6)

   (c) [5 pts.] (1 2 3) (1 2 3 4) (1 3 2)

   (c) (1 4 2 3)

9. [20 pts.] True or False. Circle one.

   (a) T $\boxed{\text{F}}$ The empty set is a group. (A group must contain an identity element.)

   (b) $\boxed{\text{T}}$ F The law of composition in a group must be associative. (By definition.)

   (c) T $\boxed{\text{F}}$ The law of composition in a group must be commutative. (S_3)

   (d) T $\boxed{\text{F}}$ A group may not be an infinite set. ($\mathbb{R}^+$ is an infinite group.)

   (e) $\boxed{\text{T}}$ F $|S_5| = 120$. ($|S_5| = 5! = 120$)

   (f) $\boxed{\text{T}}$ F If G is a group and $a \in G$, then $a^{-1} \in G$. (By definition.)

   (g) T $\boxed{\text{F}}$ If G is a group and $a \in G$, then $a^{-1} \neq a$.
   (The identity element is always its own inverse.)

   (h) $\boxed{\text{T}}$ F In every group the identity element is unique. (Proved in the notes.)

   (i) T $\boxed{\text{F}}$ According to Lagrange's theorem, if a group G has a subgroup H, then $|H|$ divides $|G|$. (G must be finite.)

   (j) T $\boxed{\text{F}}$ According to Lagrange's theorem, if G is a group and $|G| = 12$, then G has a subgroup of order 4, because 4 divides 12.
   (Converse of Lagrange's theorem is false.)

10. [5 pts.] If G is a group and $ab = ba$ for all $a, b \in G$, then G is called what?

10. **abelian**

11. [5 pts.] Is the following permutation odd or even?

$$(1\ 2\ 3\ 4)$$

11. **odd**

12. [10 pts.] Define: the kernel of a homomorphism.

Solution: The kernel of a homomorphism is the set of all elements in the domain of the homomorphism which get mapped to the identity element in the codomain.

13. [10 pts.] Let G be a group, prove that "the conjugation by g" map $\varphi_g : G \to G$ given by $\varphi_g : a \mapsto gag^{-1}$ is a homomorphism.

Solution: Let $a, b \in G$, then

$$\begin{aligned}
\varphi_g(ab) &= gabg^{-1} \\
&= ga1bg^{-1} \\
&= ga(g^{-1}g)bg^{-1} \\
&= (gag^{-1})(gbg^{-1}) \\
&= \varphi_g(a)\varphi_g(b).
\end{aligned}$$

The answers below are the class averages.

14. [5 pts.] On a scale of 1 to 10 where 1 is extremely easy and 10 is extremely hard, how hard was this test?

14. **6.1**

15. [5 pts.] On a scale of 1 to 10 where 1 is completely unfair and 10 is completely fair, how fair was this test?

15. **8.0**