Name ______________________________

1. [10 pts.] Consider the set of odd functions, $V = \{f(x) \mid f(-x) = -f(x)\}$. Show that V is closed under linear combinations.

2. (a) [6 pts.] Find the general solution, $y(x)$, to the differential equation:

$$y'' - 4y' + 4y = 0$$

   (b) [5 pts.] Give an example of a linear, homogeneous differential equation with constant coefficients that has characteristic equation: $r(r+1)^2 = 0$.

3. [10 pts.] Use the method of undetermined coefficients to find a general solution to:

$$y'' - y' - 2y = 3\cos x$$

4. For each differential equation below, determine the correct form of y_p to use in the method of undetermined coefficients.

   Do not solve for the unknown coefficients!

   (a) [5 pts.] $y'' + 9y = x^2 e^x$

   (b) [5 pts.] $y'' + 9y = \cos 3x$

   (c) [5 pts.] $y'' + 2y' + 10y = e^{-x} \cos 3x$

5. [14 pts.] True or False. Circle one.

   (a) T F If the Wronskian of three functions $W(y_1, y_2, y_3) \equiv 0$, then the three functions are linearly dependent.

   (b) T F The set of polynomials: $\{1, 1+x, 1+x+x^2\}$ is linearly dependent.

   (c) T F The Wronskian for two functions is defined to be: $W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$.

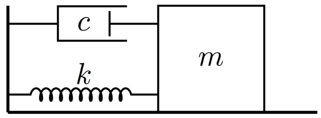
   (d) T F The set of solutions to a linear, homogeneous differential equation form a vector subspace of the set of all functions.

   (e) T F A basis for the solution space of a third order, linear, homogeneous differential equation will always contain three vectors.

   (f) T F All mass, spring, dashpot systems exhibit oscillatory behavior.

   (g) T F The natural frequency, ω_0, of a mass, spring system is given by $\omega_0 = \frac{k}{m}$.

6. Consider the mass, spring, dashpot system below, which is governed by equation (1).



$$mx'' + cx' + kx = F_0 \cos \omega t \tag{1}$$

(a) [3 pts.] Suppose $m = 1, k = 25$. What value must c have for the system to be critically damped?

(b) [3 pts.] Now suppose $m = 1, c = 1$ and $k = 100$. This system is (**circle one**):

overdamped critically damped underdamped

(c) [5 pts.] Suppose again that $m = 1, c = 1, k = 100$. What is the natural frequencey of this system, ω_0?

(d) [4 pts.] Suppose we have the same m, c, k values as part (c), with $F_0 = 2$. Recall that the amplitude of oscillation as a function of ω (the driving frequency) is given by:

$$C(\omega) = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

We can maximize the above amplitude function by differentiating with respect to ω, setting that equal to 0, and solving for ω. Upon doing so, we find:

$$\omega_\mathrm{r} = \sqrt{\frac{k}{m} - \frac{c^2}{2m^2}} \tag{2}$$

Calculate ω_r, where practical resonance will occur using formula (2).

Circle one: $\omega_\mathrm{r} < \omega_0$ $\omega_\mathrm{r} = \omega_0$ $\omega_\mathrm{r} > \omega_0$

7. [10 pts.] Use the Laplace transform method to solve the following IVP:

$$y' + 2y = e^{-t} \qquad y(0) = 0.$$

8. [15 pts.] Use the Laplace transform method to solve the following IVP:

$$y'' + 4y' - 5y = e^{-t} \qquad y(0) = 0, y'(0) = 1.$$

Function	Transform	Function	Transform
$\mathscr{L}\{y(t)\}$	$Y(s)$	$\mathscr{L}\{e^{at}\}$	$\frac{1}{s-a}$
$\mathscr{L}\{y'(t)\}$	$sY(s)-y(0)$	$\mathscr{L}\{t^n e^{at}\}$	$\frac{n!}{(s-a)^{n+1}}$
$\mathscr{L}\{y''(t)\}$	$s^2Y(s)-sy(0)-y'(0)$	$\mathscr{L}\{\cos kt\}$	$\frac{s}{s^2+k^2}$
$\mathscr{L}\{y'''(t)\}$	$s^3Y(s)-s^2y(0)-sy'(0)-y''(0)$	$\mathscr{L}\{\sin kt\}$	$\frac{k}{s^2+k^2}$
$\mathscr{L}\{e^{at}f(t)\}$	$F(s-a)$	$\mathscr{L}\{k\}$	$\frac{k}{s}$

Table 1: Table of Laplace Transforms

Page:	1	2	3	4	5	6	Total
Points:	21	10	29	15	10	15	100
Score:							