

# Final Exam Study Guide

Your final exam is on Tuesday, December 16 from 1pm to 3pm. It will be held in room BEH S 110, the same room where lectures are held. Notice this is **not** the room in which you meet for weekly labs. The final exam is designed to take about 2 hours to complete, but you may have up to 2.5 hours, that is until 3:30 pm.

The final exam is comprehensive. However, only about 1/4 (or less) of the questions will come from chapters 1-5, the rest, or 3/4 of the exam will come from the following sections 10.1–10.5, 6.1, 7.1–7.4.

You will be allowed one  $8.5 \times 11$  inch sheet of paper with notes on front and back. Also a table of necessary Laplace transforms will be provided. As usual, scientific calculators are allowed, but no graphing calculators, phone calculators, etc. will be allowed.

This study guide is your best aid in preparing for the exam, because I will only cover topics that are mentioned in this guide. In other words, if it is not mentioned in this guide, then it won't appear on your final exam!

## Chapter 1

- Know how to identify a separable, first order DE and solve it via separation of variables and integration.
- Know how to identify a linear, first order DE and solve it via the integrating factor method.

## Chapter 2

(No test questions from material in chapter 2.)

## Chapter 3

- Know how to solve linear systems such as  $A\vec{x} = \vec{b}$  via:
  - Augmentation, i.e. performing elementary row ops on  $[A | \vec{b}]$ .
  - Computing  $A^{-1}$  and then computing  $\vec{x} = A^{-1}\vec{b}$ .
- Be familiar with §3.5 Theorem 7 and §3.6 Theorem 2. (There will be at least one true/false question regarding these related theorems.)
- Know how to compute a determinant via cofactor expansion.

## Chapter 4

- Know the definitions for *linearly dependent* set of vectors and *linearly independent* set of vectors.
- Know how to determine whether a set of vectors in  $\mathbb{R}^n$  is *linearly dependent* or *linearly independent*. See §4.3 Theorem 2. If they are linearly dependent, you need to be able to find a linear combination of them equal to the zero vector.
- Know how to define: *span*, *basis* and *dimension*.

## Chapter 5

- **Very Important:** Know how to find the general solution to a homogeneous, linear DE with constant coefficients via finding the roots of the *characteristic equation*. Be able to handle all three possibilities:
  1. Distinct roots
  2. Repeated roots
  3. Complex conjugate roots (use completing the square)
- **Very Important:** Be prepared to solve a nonhomogeneous equation by the method of undetermined coefficients. I won't give you a hard problem, but there will be one of these on your final exam.

## Chapter 10

- Know the definition of the Laplace transform, and how to use it.

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

- Know how to compute partial fraction decompositions.
- **Very Important:** Know how to solve IVPs via the Laplace transform method. (You will have one homogeneous problem and one nonhomogeneous problem).
- Understand §10.3 Theorem 1 (Translation on the  $s$ -Axis) p. 636

$$\mathcal{L}\{e^{at} f(t)\} = F(s - a)$$

- Know the definition of convolution:

$$f(t) * g(t) = \int_0^t f(\tau)g(t - \tau) d\tau$$

- Know how to use the convolution theorem to solve IVPs when the Laplace transform of the right hand side of a DE is the product of two functions of  $s$ .

$$\mathcal{L}\{g(t) * h(t)\} = G(s) \cdot H(s)$$

- Know how to write a “boxcar” function via two unit step functions. This function can be used to turn on another function say  $f(t)$  at time  $a$  and then turn it off again at time  $b$ , assuming  $b > a$ .

$$[u(t - a) - u(t - b)]f(t - a)$$

- Understand §10.5 Theorem 1 (Translation on the  $t$ -Axis)

$$\mathcal{L}\{u(t - a)f(t - a)\} = e^{-as}F(s)$$

## Chapter 6

- Know the eigenvalue–eigenvector equation  $A\vec{v} = \lambda\vec{v}$ .
- Know how to compute eigenvalues by solving the eigenvalue equation  $|A - \lambda I| = 0$ .
- Know how to compute eigenvectors by solving the eigenvector equation  $(A - \lambda I)\vec{v} = \vec{0}$ .

## Chapter 7

- **Very Important:** Know how to model a three tank problem via a system of first order equations.
- **Very Important:** Know how to solve a homogeneous system of first order equations via the eigenvalue and eigenvector method.
- Be prepared to handle complex eigenvalues and eigenvectors. See my lecture notes for an explanation on how to get two real–valued eigenfunctions from the real and imaginary components of a complex–valued eigenfunction.
- **Very Important:** Know how to solve a homogenous system of second order equations via the eigenvalue and eigenvector method.
- §7.5 will **not** be covered on the final exam.