Name _____

1. [10 pts.] Find the general solution to

$$y'' + 2y' + 10y = 0.$$

2. [10 pts.] Determine the linear, homogeneous differential equation with constant coefficients which has solutions:

$$y_1 = e^{-x} \quad y_2 = e^{x} \quad y_3 = xe^{x}.$$

Hint: What would the roots of the characteristic equation need to be in order to get these solutions?

3. [15 pts.] Use Gauss-Jordan elimination to solve the system

$$\begin{aligned} x_1 + \ \ x_2 + \ \ x_3 + \ \ x_4 &= 12 \\ x_2 - \ \ x_3 + 4x_4 &= \ \ 5 \\ 3x_1 + 2x_2 + 4x_3 - \ \ x_4 &= 31 \end{aligned}$$

4. [5 pts.] The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ projects any vector in $\mathbb{R}^3$ to the xy–plane of $\mathbb{R}^3$, because

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}.$$

In other words, A sets the z component of any vector to 0. What is the solution space of the following homogeneous equation?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

A. All vectors in the xy–plane

B. All vectors in the xz–plane

C. All vectors in the yz–plane

D. Only the zero vector

E. $\mathbb{R}^3$

F. The z–axis

5. [5 pts.] Recall that equation (1) above is exactly equivalent to the equation

$$x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

What is the span of the three column vectors of matrix A?

A. All vectors in the xy–plane

B. All vectors in the xz–plane

C. All vectors in the yz–plane

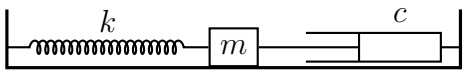
D. Only the zero vector

E. $\mathbb{R}^3$

F. The z–axis

6. [15 pts.] Determine the correct differential equation for the free (undriven), mass–spring–dashpot system pictured and solve it for $x(t)$.

   Assume: $m = 1,\ c = 2,\ k = 10,\ x(0) = 5,\ x'(0) = 0.$



7. [10 pts.] Circle or box the letter corresponding to the correct trial particular solution, y_p, for each given equation.

(a) $y'' - y' - 6y = 2 + x$

A. $y_p = Ax$
B. $y_p = A + Bx$
C. $y_p = Ae^{-2x} + Be^{3x}$
D. $y_p = (A + Bx)e^x$

(b) $y'' + 3y' + 2y = e^{-x}$

A. $y_p = Ae^{-x}$
B. $y_p = Axe^{-x}$
C. $y_p = Ae^{-x} + Be^{-2x}$
D. $y_p = x(Ae^{-x} + Be^{-2x})$

(c) $y'' + 3y' + 2y = xe^{4x}$

A. $y_p = Ae^{4x}$
B. $y_p = Axe^{4x}$
C. $y_p = (A + Bx)e^{4x}$
D. $y_p = (A + Bx + Cx^2)e^{4x}$

(d) $y'' + 9y = 5\cos 3x$

A. $y_p = A\cos 3x$
B. $y_p = (A + Bx)\cos 3x$
C. $y_p = A\cos 3x + B\sin 3x$
D. $y_p = x(A\cos 3x + B\sin 3x)$

8. [20 pts.] True or False. Circle one.

(a) T F A system of linear, algebraic equations may have two and only two distinct solutions.

(b) T F A homogeneous system of linear algebraic equations always has at least one solution.

(c) T F The solution space of a homogeneous, linear differential equation is a vector subspace of $\mathcal{F}$, the vector space of all real valued functions.

(d) T F Multiplying a row of a matrix by any real number is a valid elementary row operation.

(e) T F $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{vmatrix} = 6$

(f) T F The following equation of a mass—spring—dashpot system is critically damped.

$$x'' + 5x' + \frac{25}{4}x = 0$$

(g) T F Let the set B be a basis for the vector space V. If you remove any vector from B then it will no longer span V.

(h) T F Let the set B be a basis for a vector space V. If you add any vector to B, then its vectors will be linearly dependent.

(i) T F The matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 9 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

is in reduced row–echelon form.

(j) T F Suppose a square matrix, A has determinant, $|A| = 5$. If we interchange the first two rows of A to make A', then $|A'| = -5$.

9. The are two ways you can fail to earn the points for the last two questions:

   1. not providing an answer,
   2. not writing an integer between 1 and 10 inclusive.

   (a) [5 pts.] Rate the difficulty of this exam on a scale of 1 to 10, where 1 is very easy and 10 is very difficult.

   (a) _____

   (b) [5 pts.] Rate the fairness of this exam on a scale of 1 to 10, where 1 is completely unfair and 10 is completely fair.

   (b) _____

Scratch Paper

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	15	5	5	15	10	20	10	100
Score:										