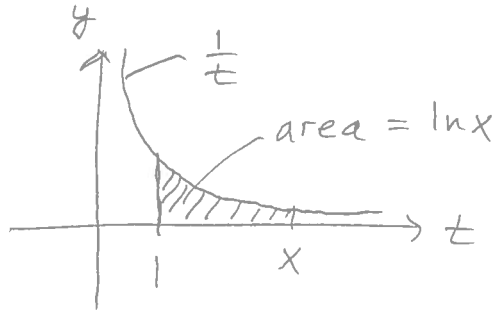
Exam 1 Study Guide

6.1

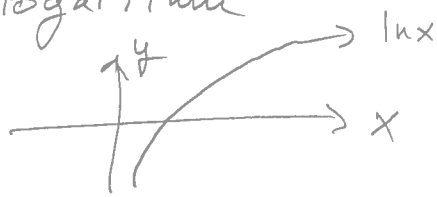
* Definition of $\ln(x)$ as accumulation function and 1st Fundamental Theorem of Calculus.

$$\ln(x) = \int_1^x \frac{1}{t}\,dt$$



1st FTC $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x) \implies \frac{d}{dx}[\ln x] = \frac{1}{x}$

* Properties of the logarithm
* Graph of $\ln(x)$



* Logarithmic differentiation

6.2

* <u>Thm</u> If f is strictly monotonic on its domain then f has an inverse

* Properties of functions and their inverses

$$x = f^{-1}(y) \iff y = f(x)$$

* $(f \circ f^{-1})(x) = x$ <u>and</u> $(f^{-1} \circ f)(x) = x$

* Be able to <u>graph</u> a function and its inverse
* Inverse function theorem

6.3

* Properties of e^x: ① $e^a e^b = e^{(a+b)}$

② $\dfrac{e^a}{e^b} = e^{(a-b)}$

* $\boxed{D_x\left[e^u\right] = e^u \cdot u'}$ $\boxed{\int e^u du = e^u + C}$

chain rule

* Definition of e^x as inverse of $\ln x$, i.e.

$$y = \ln x \iff e^y = x$$

So $e^{\ln x} = x$ and $\ln(e^x) = x$

6.4

* Definitions

$$\boxed{\begin{aligned} a^x &= e^{(x \cdot \ln a)} \\ \log_a x &= \frac{\ln x}{\ln a} \end{aligned}}$$

* Thm B

$$\boxed{\begin{aligned} \frac{d}{dx}\left[a^x\right] &= a^x \cdot \ln a \\ \int a^x dx &= \frac{1}{\ln a} \cdot a^x + C \end{aligned}}$$

* Inverse Property $\boxed{y = \log_a x \iff x = a^y}$

6.4 (cont.)

* $\frac{d}{dx}\left[\log_a x\right] = \frac{d}{dx}\left[\frac{\ln x}{\ln a}\right] = \frac{1}{\ln a}\cdot\frac{1}{x}$

$$\int \log_a x \, dx = \int \frac{\ln x}{\ln a}\, dx = \frac{1}{\ln a}\int \ln x \, dx = ?$$

Don't know this yet!

6.5

* Population Model

$$\boxed{\underset{\text{(model)}}{\frac{dP}{dt} = kP} \implies \underset{\text{(solution)}}{P(t) = P_0 e^{kt}}}$$

know how to derive the solution by separation of variables, i.e.

$$\int \frac{dP}{P} = \int k\,dt \implies \ln|P| = kt + C$$

$$\implies P(t) = P_0 e^{kt}$$

* Know how to use this to model exponential growth and decay.

* NO Newton cooling problems

* NO diffusion model problems

6.6

* Solving linear equations by the integrating factor technique:

1. Put in std form: $\frac{dy}{dt} + p(t)y = g(t)$
2. $I = e^{\int p(t)\,dt}$
3. Multiply <u>both</u> sides of eqn. by I,
4. The left hand side will always be

$$\frac{d}{dt}\left[y \cdot e^{\int p(t)\,dt}\right]$$

5. Integrate both sides
6. Solve for y!

* Know how to set up tank problem equations and how to solve them.