Exam 1 Study Guide

6.1 * Definition of $\ln(x)$ as accumulation function and 1st Fundamental Theorem of Calculus.

$$\ln(x) = \int_{1}^{x} \frac{1}{t} \, dt$$

* 1st FTC $\frac{d}{dx} \left[ \int_{a}^{x} f(t) \, dt \right] = f(x) \implies \frac{d}{dx} [\ln x] = \frac{1}{x}$

* Properties of the logarithm

* Graph of $\ln(x)$

* Logarithmic differentiation

6.2 * Theorem: If $f$ is strictly monotonic on its domain then $f$ has an inverse

* Properties of functions and their inverses

$x = f^{-1}(y) \iff y = f(x)$

* $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

* Be able to graph a function and its inverse

* Inverse function theorem
6.3
\[ \text{Properties of } e^x: \]
1. \[ e^a e^b = e^{a+b} \]
2. \[ \frac{e^a}{e^b} = e^{a-b} \]

\[ \text{Proof: Chain rule} \]

\[ \text{Definition of } e^x \text{ as inverse of } \ln x, \text{i.e.,} \]

\[ y = \ln x \quad \iff \quad e^y = x \]

So \[ e^{\ln x} = x \] and \[ \ln (e^x) = x \]

6.4
\[ \text{Definitions} \]

\[ a^x = e^{(x \cdot \ln a)} \]

\[ \log_a x = \frac{\ln x}{\ln a} \]

\[ \text{Thm B} \]

\[ \frac{d}{dx} [a^x] = a^x \cdot \ln a \]

\[ \int a^x \, dx = \frac{1}{\ln a} \cdot a^x + C \]

\[ \text{Inverse Property} \]

\[ y = \log_a x \quad \iff \quad x = a^y \]
6.4 (cont.)
\[ \frac{d}{dt} \left[ \log_a x \right] = \frac{d}{dx} \left[ \frac{\ln x}{\ln a} \right] = \frac{1}{\ln a} \cdot \frac{1}{x} \]

\[ \int \log_a x \, dx = \int \frac{\ln x}{\ln a} \, dx = \frac{1}{\ln a} \left( \int \ln x \, dx \right) = ? \]

Don't know this yet!

6.5

* Population Model

\[ \frac{dP}{dt} = kP \quad \Rightarrow \quad P(t) = P_0 e^{kt} \]

(model) (solution)

Know how to derive the solution by separation of variables, i.e.

\[ \int \frac{dP}{P} = \int k \, dt \quad \Rightarrow \quad \ln |P| = kt + C \]

\[ \Rightarrow \quad P(t) = P_0 e^{kt} \]

Know how to use this to model exponential growth and decay.

* NO Newton cooling problems

* NO diffusion model problems
6.6

Solving linear equations by
the integrating factor technique:

1. Put in std form: \( \frac{dy}{dt} + p(t)y = g(t) \)

2. \( I = e^{\int p(t) dt} \)

3. Multiply both sides of eqn. by \( I \).

4. The left hand side will always be

\[ \frac{d}{dt} \left[ y \cdot e^{\int p(t) dt} \right] \]

5. Integrate both sides

6. solve for \( y \! \! \)!

* Know how to set up tank problem equations and how to solve them,