5.2 Simple Interest, Compound Interest and APY

**Arithmetic Sequence**

- recursive: \( a_n = a_{n-1} + d \)
- iterative: \( a_n = a_1 + (n-1)d \)

**Simple Interest**

\[ S = P + t \cdot (P \cdot r) = P(1 + rt) \]

**Geometric Sequence**

- recursive: \( a_n = d \cdot a_{n-1} \)
- iterative: \( a_n = a_1 \cdot d^{n-1} \)

**Compound Interest**

\[ S = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ S = S(t) = \text{value of account after } t \text{ years.} \]

\[ P = \text{principal or the initial investment} \quad (P = a_1) \]

\[ r = \text{interest rate} \]

\[ n = \# \text{ of compounding periods per year} \]

**Def.** interest accrued/earned is defined to be

\[ I = S - P \]

**Ex. 1** Invest $10,000 at 7.5% for four years.

a) **Simple Interest**

\[ S = 10,000 \left(1 + (0.075)\right)^4 \]
\[ = 10,000(1.3) \]
\[ = 13,000 \]

b) **Compound Interest** (compounded twice yearly, \( n = 2 \))

\[ S = 10,000 \left(1 + \frac{0.075}{2}\right)^{2 \cdot 4} \]
\[ = 10,000 \left(1 + 0.0375\right)^8 \]
\[ = 13,424.71 \]
Ex 2. Mary borrowed $3,000 at an interest rate of 18%. How much interest will accrue after 0.5 weeks?

a) simple interest

\[
0.5 \text{ weeks} \times \left( \frac{1 \text{ yr}}{52 \text{ weeks}} \right) = 0.095238 \text{ yr.} = t
\]

\[
I = S - P = P(1 + rt) - P = P + Prt - P = Prt = (3,000)(0.18)(0.095238) = 675
\]

b) compounded quarterly interest

\[
I = S - P = P \left( 1 + \frac{r}{n} \right)^{nt} - P = 3,000 \left( 1 + \frac{0.18}{4} \right)^{4 \times 0.095238} - 3,000 = 3,000 \left( 1.045 \right)^{5} - 3,000 = 3,738.55 - 3,000 = 738.55
\]

Ex 4. How much do you need to invest now in order to retire with $2,000,000 in 40 years if interest compounds monthly at 8%? (Ans. P = $82,394.77)

\[
S = P \left( 1 + \frac{r}{n} \right)^{nt}
\]
**Annual Percentage Yield (APY)**

APY is a tool that allows you to compare the return from different interest bearing accounts. It allows you to compare apples to apples.

\[ \text{APY} = (1 + \frac{r}{n})^n - 1 \]  
**periodic compounding**

\[ \text{APY} = e^r - 1 \]  
**continuous compounding**

Ex. 6  Which is the best investment deal in the long run?

a) 10% simple interest

b) 9% compounded annually

c) 8.75% **quarterly**

d) 8.6% **continuously**