# 4.4 Properties of Logarithms

Recall : $\boxed{\log_a x = y \iff a^y = x \qquad a>0,\ a\neq 1}$

This is how we define the log function. Memorize!!

## Properties of the logarithm

Follow from the definition

1. $\log_a 1 = 0$ — This says that the point $(1,0)$ [$(x,y)$] is always on the graph of the log function. (Follows from $a^0 = 1$.)

   $1$ ↑ input, $0$ ↑ output

2. $\log_a a = 1$ $\Rightarrow$ $(a,1)$ is always on the graph because $a^1 = a$.

   input ↓ $a$, output ↓ $1$

Memorize

3. $\log_a a^x = x$ — This says that log is the inverse of exponentiation.

4. $a^{\log_a x} = x$ — This says that exponentiation is the inverse of logarithm.

5. $\log_a mn = (\log_a m) + (\log_a n)$

   "The log of a product is the sum of the logs."

6. $\log_a \frac{m}{n} = (\log_a m) - (\log_a n)$

   "The log of a quotient is the difference of the logs."

7. $\log_a m^n = n \cdot \log_a m$

   "You can pull exponents out to the front."

Use logarithm properties to expand these expressions:

a) $\log_2\left(\frac{3x}{x+4}\right)$

b) $\log\left(y^4\sqrt{y-3}\right)$

c) $\ln\left((2x+1)(x-5)\right)$

d) $\log_3\left(\frac{x^2y^3}{w^4}\right)$

Use logarithm properties to condense these expressions:

a) $\log_5 8 - \frac{1}{3}\log_5 2$

b) $\log_4(3x+1) + 5\log_4(x-2)$

c) $3(\ln 6 + 4\ln 5 - \ln 2)$

d) $\log 5x + 2(\log x + \log(x+2))$

Evaluate these expressions exactly (without a calculator).

a) $\log_2 \sqrt{2} + \log_3 \sqrt[3]{3} - \log_4 \sqrt[4]{4}$

b) $\log_7 \left(7\sqrt{7}\right)^3$

c) $\ln e^{6.23} - \log 100 + \log_5\left(\frac{1}{125}\right)$