4.4 Properties of Logarithms

Recall: \[ \log_a x = y \iff a^y = x \quad a > 0, \ a \neq 1 \]

This is how we define the log function. Memorize!!

Properties of the logarithm

Follow from the definition

1. \[ \log_a 1 = 0 \]
   
   \[ \begin{array}{c}
   \text{input} \\
   \uparrow \\
   \text{output}
   \end{array} \]

   This says that the point \((1, 0)\) is always on the graph of the log function. (Follows from \(a^0 = 1\)).

2. \[ \log_a a = 1 \]

   \[ (a, 1) \text{ is always on the graph because } a^1 = a. \]

3. \[ \log_a a^x = x \]

   This says that log is the inverse of exponentiation.

4. \[ a \log_a x = x \]

   This says that exponentiation is the inverse of logarithm.

Memorize

5. \[ \log_a mn = (\log_a m) + (\log_a n) \]

   "The log of a product is the sum of the logs."

6. \[ \log_a \frac{m}{n} = (\log_a m) - (\log_a n) \]

   "The log of a quotient is the difference of the logs."

7. \[ \log_a m^n = n \cdot \log_a m \]

   "You can pull exponents out to the front."
Use logarithm properties to expand these expressions:

a) \( \log_2 \left( \frac{3x}{x+4} \right) \)

b) \( \log \left( y^{\sqrt{y-3}} \right) \)

c) \( \ln \left( (2x+1)(x-5) \right) \)

d) \( \log_3 \left( \frac{x^2y^3}{w^4} \right) \)

Use logarithm properties to condense these expressions:

a) \( \log_5 8 - \frac{1}{3} \log_5 2 \)

b) \( \log_4 (3x+1) + 5 \log_4 (x-2) \)

c) \( 3 \left( \ln 6 + 4 \ln 5 - \ln 2 \right) \)

d) \( \log 5x + 2 \left( \log x + \log (x+2) \right) \)
Evaluate these expressions exactly (without a calculator).

a) \( \log_2 \sqrt{2} + \log_3 \sqrt[3]{3} - \log_4 \sqrt[4]{4} \)

b) \( \log_7 (7 \sqrt[7]{7})^3 \)

c) \( \ln e^{6.23} - \log 100 + \log_5 (\frac{1}{125}) \)