# 3.5 Graphing Polynomial Functions + Piecewise Functions

A <u>polynomial function</u> is a function that can be written with form:

degree

represents all intermediate terms

$$f(x) = a_n x^{n} + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

leading coefficient

coefficients

In this section we study the properties of polynomials which determine the shape of their graphs. First, lets review what we already know about linear (degree=1) polynomials and quadratic (degree=2) polynomials.

<u>Linear:</u> $f(x) = a_1 x + a_0 = mx + b$

slope

y-intercept (so named because $f(0) = a_0$)

$a_1 < 0 \Rightarrow$ <u>decreasing</u> or downward pointing line.

$a_1 > 0 \Rightarrow$ <u>increasing</u> or upward " ".

<u>Quadratic:</u> $f(x) = a_2 x^2 + a_1 x + a_0 = ax^2 + bx + c$

$a_2 < 0 \Rightarrow$ <u>concave down</u>

$a_2 > 0 \Rightarrow$ <u>concave up</u>

y-intercept

How about polynomials of higher degree? There are a few <u>general principles</u> that the graphs of higher degree polynomials follow:

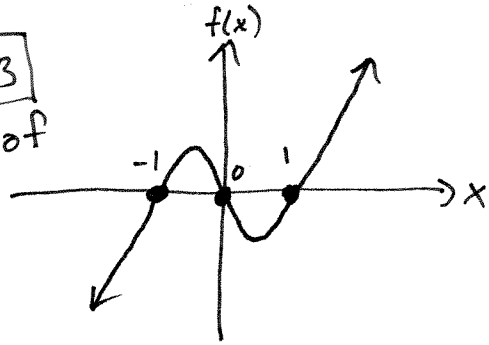
<u>Principle 1</u> $\boxed{\text{degree} - 1 = \text{maximum \# of direction changes}}$

<u>For example</u>:

$f(x) = x^3 - x = (x-0)(x-1)(x+1)$

$\boxed{\text{degree} = 3}$

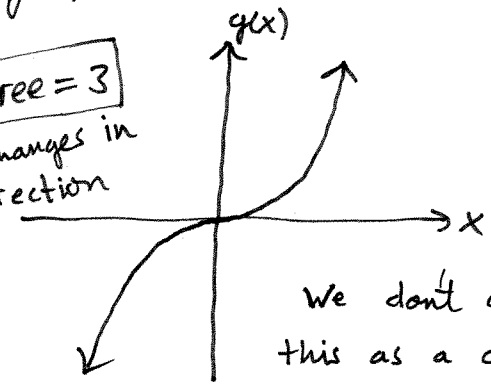
2 changes of direction




$g(x) = x^3 = (x-0)(x-0)(x-0) = (x-0)^3$

$\boxed{\text{degree} = 3}$

0 changes in direction




We don't count this as a change in direction because the slope never changes from positive to negative.

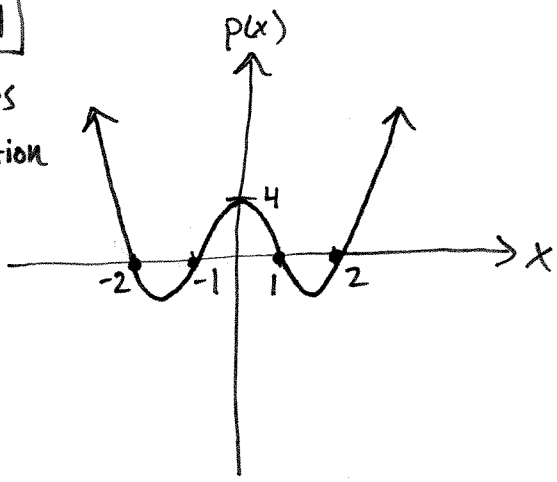
$p(x) = x^4 - 5x^2 + 4$

$= (x^2-1)(x^2-4)$

$= (x+1)(x-1)(x+2)(x-2)$

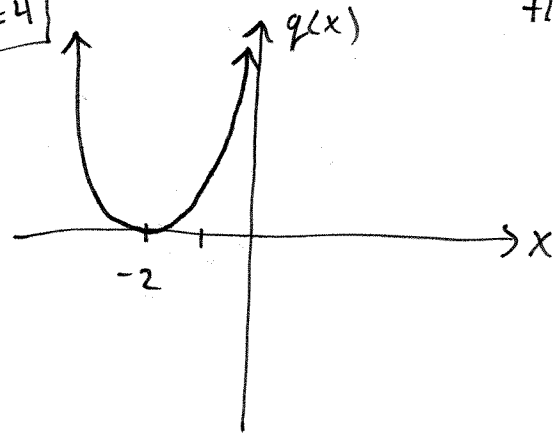
$\boxed{\text{degree} = 4}$

3 changes of direction



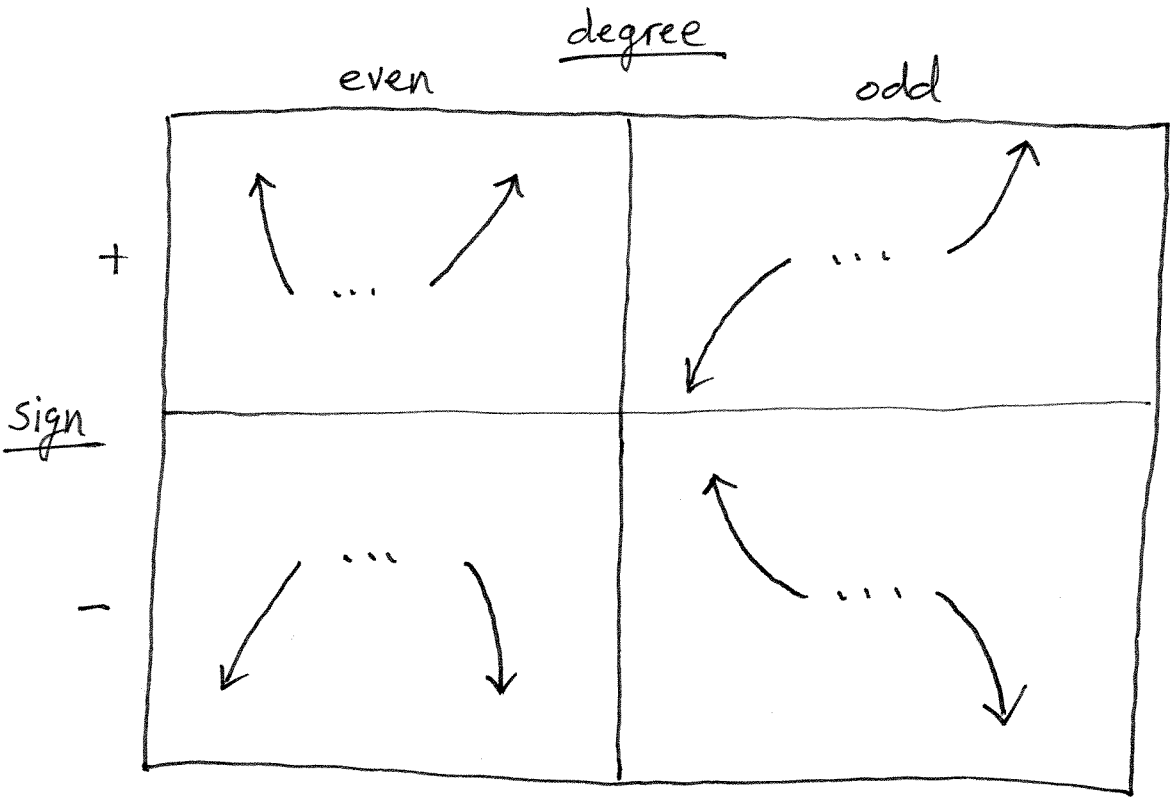

$q(x) = (x+2)^4 = x^4 + 8x^3 + 24x^2 + 32x + 16$

$\boxed{\text{degree} = 4}$



**Principle 2**

> The sign of the leading coefficient and the evenness/oddness of the degree determine the polynomial's behavior at $\pm\infty$.

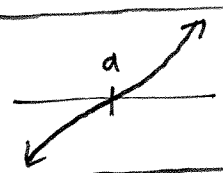
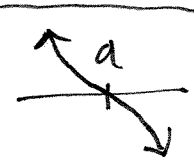
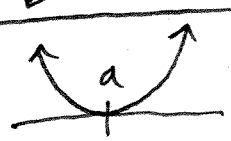
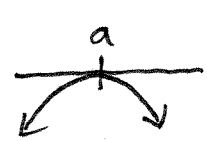



(Here the "..." represents wiggles or changes in direction.)

**Principle 3**

> If a polynomial factors, then
> 1) factors raised to an <u>odd</u> power <u>cross</u> the x-axis.
> 2) " " " " <u>even</u> " <u>kiss</u> " " .

Let $n \in \mathbb{N}$, then $2n$ is even and $2n+1$ is odd.

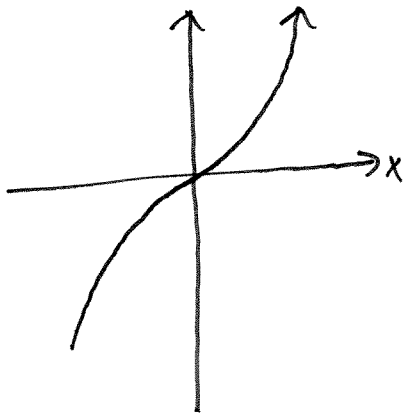
| | | | | |
|---|---|---|---|---|
| odd | $(x-a)^{2n+1}$ |   | OR   | cross |
| even | $(x-a)^{2n}$ |   | OR   | kiss |

## Principle 4

> The higher the power of a factor, the flatter the crossing or kissing becomes.

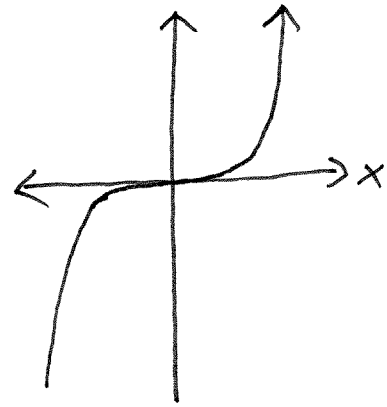
**Crossings**

$x^3 = (x-0)^3$



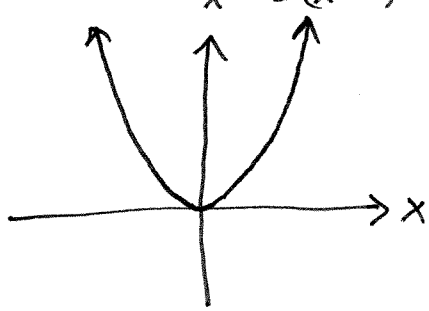

vs.

$x^7 = (x-0)^7$



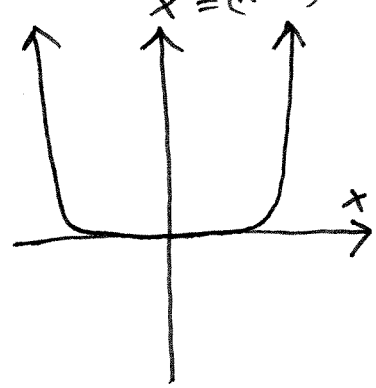

**Kissings**

$x^2 = (x-0)^2$



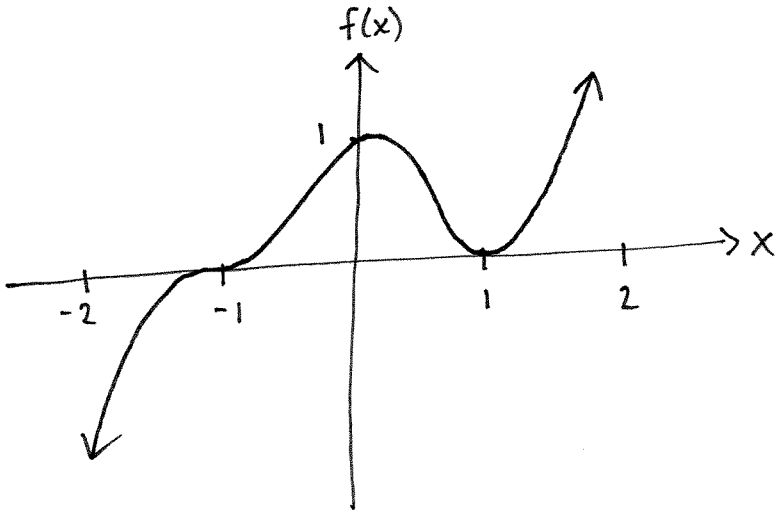

vs.

$x^{10} = (x-0)^{10}$




**Example:** $f(x) = x^5 + x^4 - 2x^2 + x + 1$ standard form

$f(x) = (x+1)^3 \cdot (x-1)^2$ factored form



## Piecewise Functions

Cut the real number line $\mathbb{R}$ into pieces, and define the function however you wish on each piece.

**Example**

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

pieces of $\mathbb{R}$.

function values

Q: what is the common name for $f(x)$ above?

**Example**

$$g(x) = \begin{cases} 2x+1 & x < 1 \\ 4 & 1 \leq x < 2 \\ 3 & x \geq 2 \end{cases}$$

graph: