3.2 Functions

Def. A relation is a set of ordered pairs \((x, y)\), where \(x\) and \(y\) are any mathematical objects, usually numbers, but they could be sets, vectors, functions etc.

**Examples**
1) \[\{(0, 0), (1, 1), (2, 4), (3, 9), (4, 16)\}\]
2) \[\{(2, 4), (1, 3), (3, 2), (2, 3), (4, 5)\}\]
3) \[\{([1], [2]), ([1], [2]), ([0], [1])\}\]

Def. The first element of each ordered pair is called an input and the second element of each ordered pair is called an output.

Def. A function is a relation where every input is related to exactly one output.

Q: Which of the above relations are functions?

Def. The domain of a function is the set of allowable inputs, i.e. the set of points for which the function is defined. We don't allow: 1) division by zero 2) taking even roots of negative numbers.

Def. The range of a function is the set of outputs. e.g. \(\sqrt{-3}\)
14.) \( y = \sqrt{7x+1} \) function? Yes!

Domain? This is a square root, which is an even root, (we could have written \( \sqrt{7x+1} \)) thus we can't let the values inside the square root become negative.

\[ 7x+1 \geq 0 \Rightarrow 7x \geq -1 \Rightarrow x \geq \frac{-1}{7} \]

domain of \( \text{func} = [-\frac{1}{7}, \infty) \)

range "" = \([0, \infty)\)

15.) \( 3x^2 + 4y^2 = 7 \)  \(\Rightarrow\) \( 4y^2 = -3x^2 + 7 \)

\[ \Rightarrow \quad y^2 = \frac{-3}{4}x^2 + \frac{7}{4} \]

\[ y = \pm \sqrt{\frac{-3x^2 + 7}{4}} = \pm \frac{\sqrt{-3x^2 + 7}}{2} \]

This \( \pm \) makes it not a function!

Here is the equation of a circle: \( x^2 + y^2 = r^2 \)

Notice the similarity in the equations.
16.) \( y = \frac{x+1}{x-2} \) function?

\[ \text{domain} = \]

\[ \text{range} = \]

57.) \( y = \frac{1}{x^2 - 5x + 6} = \frac{1}{(x-2)(x-3)} \)

\[ \text{domain} = \]

\[ \text{range} = \]

Find the domain of the following functions:

61.) \( f(x) = \sqrt{x-5} \)

63.) \( h(x) = \frac{x^4 - 1}{x^3 - 8} \)
Evaluate each function at the given input value:

46) \( f(x) = 9 - 7x \) with \( x = 2u - \sigma \)

\[
f(2u - \sigma) = 9 - 7(2u - \sigma) = 9 - 14u + 42
\]

\[
= -14u + 51
\]

47) \( f(x) = x^2 - x + 7 \) with \( x = 8 - 2a \)

\[
f(8 - 2a) =
\]