2.5 Applications of Matrices

Suppose we have the following system of equations:

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

which is equivalent to the single matrix equation

$$\overset{A}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \overset{\vec{x}}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \overset{\vec{b}}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}$$

which we often write in shorthand: $A\vec{x} = \vec{b}$,

then we now have four ways to solve for x, y & z.

1.) Substitution §1.4
2.) Elimination §1.4
3.) Do Gauss-Jordan elimination on the augmented matrix $[A \mid \vec{b}]$. §2.3
4.) Do Gauss-Jordan elimination on the augmented matrix $[A \mid I]$ (where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$) to find A^{-1}, then apply it to $\vec{b}$, i.e. $\vec{x} = A^{-1}\vec{b}$. §2.4

Methods 1 – 3 all require about the same amount of work, but method 4 requires more work.

However, there are two reasons for choosing method 4,

a) If you have some tool (e.g. calculator, Maple, the Pivot Gauss-Jordan web site from last lecture) to compute A^{-1} for you, then all you have to do is: $A^{-1}\vec{b}$ which is just matrix multiplication.

b) If you have to solve the matrix equation $A\vec{x} = \vec{b}$ several times, but for lots of different $\vec{b}$ vectors. (Decoding messages is one example mentioned in the book.)

Feel free to use any tool you like to invert matrices (i.e. to find A^{-1}) for this section's homework. But I would still do a few by hand for the practice. (Remember you'll have to do it by hand on the exam!)

9.) A school sold, cakes, pies and boxes of cookies at a bake sale. Cakes were \$3.50 each, pies were \$5.00 each and the boxes of cookies sold for \$2.00 each. Altogether 122 items were sold. The pies sold the best, selling 8 more pies than cakes and boxes of cookies combined. The total sales were \$505. How many of each item were sold?

Solution: c = # cakes

p = # pies

b = # boxes of cookies

33.) Find three numbers that sum to 42.
The sum of the two largest is two more than four times the smallest number.
The largest number less the smallest number is two more than the middle number.

Solution:

s = smallest number

m = middle "

l = largest "

$$\left.\begin{aligned} s + m + l &= 42 \\ m + l &= 4s + 2 \\ l - s &= m + 2 \end{aligned}\right\} \Rightarrow \begin{aligned} s + m + l &= 42 \\ -4s + m + l &= 2 \\ -s - m + l &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 42 \\ -4 & 1 & 1 & 2 \\ -1 & -1 & 1 & 2 \end{array}\right] R_3 + R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 42 \\ -4 & 1 & 1 & 2 \\ 0 & 0 & 2 & 44 \end{array}\right] \tfrac{1}{2} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 42 \\ -4 & 1 & 1 & 2 \\ 0 & 0 & 1 & 22 \end{array}\right] \begin{array}{l} R_1 - R_3 \\ R_2 - R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 20 \\ -4 & 1 & 0 & -20 \\ 0 & 0 & 1 & 22 \end{array}\right] R_2 + 4R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 20 \\ 0 & 5 & 0 & 60 \\ 0 & 0 & 1 & 22 \end{array}\right] \tfrac{1}{5} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 20 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 22 \end{array}\right] R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 22 \end{array}\right] \Rightarrow \begin{aligned} s &= 8 \\ m &= 12 \\ l &= 22 \end{aligned}$$

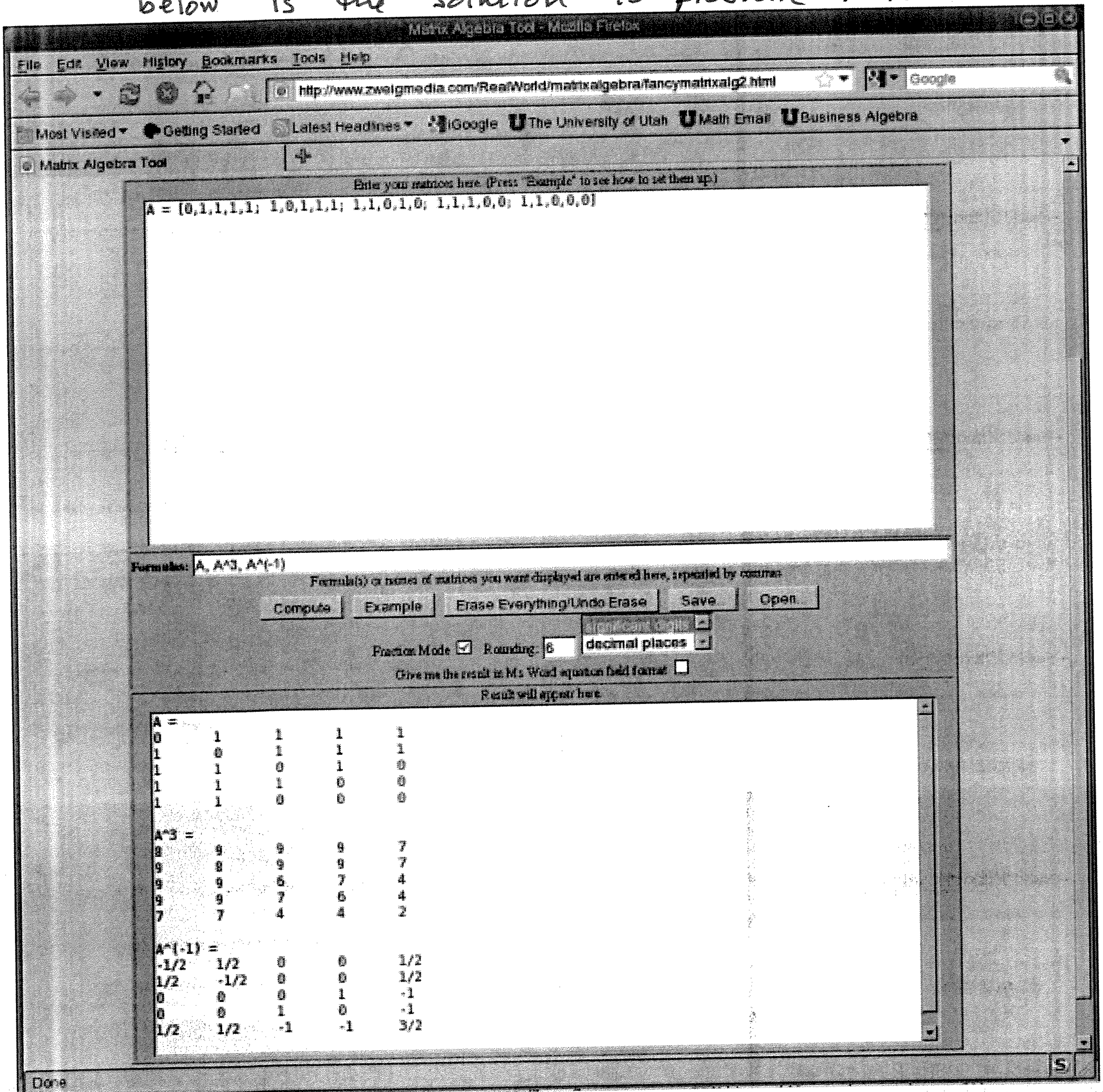
Check:

$8 + 12 + 22 = 42$ ✓

$-4(8) + 12 + 22 = 2$ ✓

$-8 - 12 + 22 = 2$ ✓

The following matrix algebra tool at www.zweigmedia.com/RealWorld/matrixalgebra/fancymatrixalg2.html can help with everything for this next assignment below is the solution to problem #27!



To compute A^{-1} enter: A^(-1) in the "Formulas:" box.