1.6 Linear Inequalities in Two Variables

Def.

Linear inequality: An inequality that can be written in the form:
- \( ax + by < c \) OR
- \( ax + by > c \) OR
- \( ax + by \leq c \) OR
- \( ax + by \geq c \).

Linear system of inequalities: A set of two or more linear inequalities in two or more variables.

Solution set: The set of all points that satisfy all inequalities simultaneously.

Half-plane: The solution set of a single linear inequality in two variables.

E.g.

Test point: A randomly chosen point used to determine whether or not a region satisfies a linear system of inequalities.
2. 9) Ex. 1 Graph the solution set of $\frac{1}{2}x - \frac{1}{2}y \leq 1$.

**Step 1** Solve inequality for $y$:

\[
\begin{align*}
-\frac{1}{2}x - \frac{1}{2}y & \leq 1 \\
\Rightarrow \quad -\frac{1}{2}y & \leq \frac{1}{2}x + 1 \\
\Rightarrow \quad -2\left[-\frac{1}{2}y \leq \frac{1}{2}x + 1\right] & \\
\Rightarrow \quad y & \geq -x - 2
\end{align*}
\]

Recall: when multiplying an inequality by a negative number we flip the inequality.

**Step 2** Graph the line which corresponds:

i.e. graph $y = -x - 2$

\[\text{y-intercept} \quad \text{slope} = -1\]

**Step 3** Shade appropriate region:

shade above if $y \geq -x - 2$, shade below if $y \leq -x - 2$
Ex 2  System of Linear Inequalities

27. \[ \begin{align*}
  2y &> -2 \quad (1) \\
  3x - 2y &< -7 \quad (2) \\
  3x - 4y &> -11 \quad (3)
\end{align*} \]

Solution:

\[ 2y > -2 \implies y > -1 \]
\[ 3x - 2y < -7 \implies -2y < -3x - 7 \implies y > \frac{3}{2}x + \frac{7}{2} \]
\[ 3x - 4y > -11 \implies -4y > -3x - 11 \implies y < \frac{3}{4}x + \frac{11}{4} \]

check with test-point:

\((-3, 0)\) ?

(1) \[ 2(0) > -2 \checkmark \]
(2) \[ 3(-3) - 2(0) < -7 \]
\[-9 < -7 \checkmark \]
(3) \[ 3(-3) - 4(0) > -11 \]
\[-9 > -11 \checkmark \]
Ex. 3

34.) A factory can manufacture 1,000 vehicles, either cars or trucks, per day. The factory must manufacture at least 200 more cars than trucks. Find the inequalities which represent this situation and then graph the solution set.