1.2 Linear Inequalities in One Variable

Def A linear inequality is an inequality that can be written in the form:

$$ax + b \leq c \quad \text{OR} \quad ax + b \geq c$$

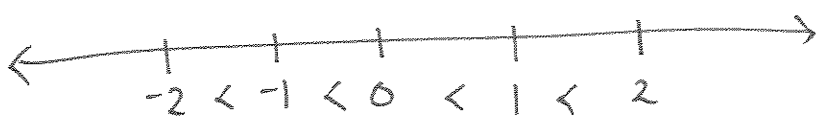
where $a \neq 0$, and $a, b, c \in \mathbb{R}$.

Linear inequalities are solved exactly the same way that linear equations are solved with one exception.

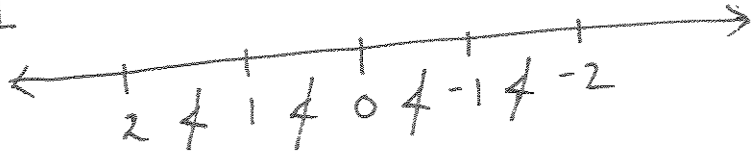
Recall: When solving a linear equation, we can:

1. Add any real number to both sides of the equation.
2. Multiply both sides of an equation by any nonzero real number.

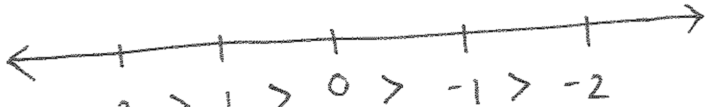
If we look at a small portion of the number line, we see that the integers ($\mathbb{Z}$) as well as the reals ($\mathbb{R}$) have a natural ordering:



However, when we multiply every number by -1, Ex 1

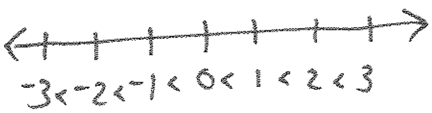


each inequality becomes false. To correct the situation, we must flip the direction of each inequality:

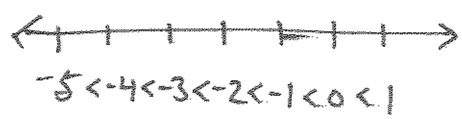


Notice that if we add a negative or positive number to each number we just shift the number line, but the original inequalities will still be true:

Ex 2

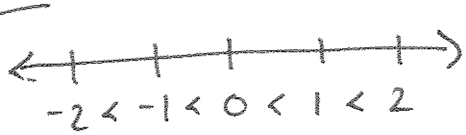


$-2 \Longrightarrow$

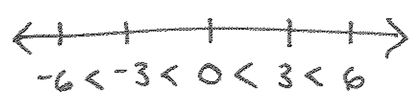


If we multiply by a positive number, the original inequalities still hold: (we're stretching the number line)

Ex 3



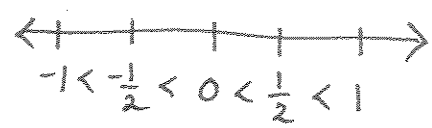
$\times 3 \Longrightarrow$



Even positive fractions maintain the original inequalities:

Ex 4

$\times \frac{1}{2} \Longrightarrow$



contraction

When solving a linear inequality, we can:

1. Add any real number to both sides of the inequality.
2. Multiply both sides of an inequality by a positive real number.
3. Multiply both sides of an inequality by a negative real number and flip the inequality.

Ex. Solve and graph the solution of:

$$\begin{array}{rcl} 4 - 6x & < & 2 \\ -2 & & -2 \end{array}$$

$$\begin{array}{rcl} 2 - 6x & < & 0 \\ +6x & & +6x \end{array}$$

$$\frac{2}{6} < \frac{6x}{6}$$

$$\boxed{\frac{1}{3} < x}$$

OR

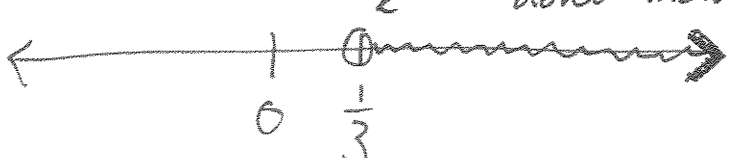
$$\begin{array}{rcl} 4 - 6x & < & 2 \\ -4 & & -4 \end{array}$$

$$\frac{-6x}{-6} < \frac{-2}{-6}$$

$$\boxed{x > \frac{1}{3}}$$

Notice in this solution, since we divided by -6 or multiplied by $\frac{-1}{6}$, we had to flip the inequality.

Solution graph:



open circle means we don't include this point in the solution set.

Ex (Problem 52)

$\underbrace{\text{Five more than}}$ $\underbrace{\text{four times a number}}$ $\underbrace{\text{is at least 21.}}$

What range of values can this number have?

Solution: Let x represent the unknown number(s).

$$\boxed{5 + 4x \geq 21}$$
$$-5 \qquad\qquad -5$$

$$\frac{4x}{4} \geq \frac{16}{4} \implies \cancel{\boxed{x \geq 16}}$$

oops!

$$\boxed{x \geq 4}$$

Ex (Problem 61)

If the revenue function is $R(x) = 40x$ and the cost function is $C(x) = 20x + 1600$, how many items must be sold to realize a profit?

Ex. (problem 69)

You are offered a job with two options:

(a) A salary of \$45,000 per year OR

(b) A salary of \$2,500 per month plus a commission ~~on gross~~ of 6% on gross sales.

For what range of ~~salaries~~ sales is the second plan better than the first?