1.2 Linear Inequalities in One Variable

Def A linear inequality is an inequality that can be written in the form:

\[ ax + b \leq c \quad \text{OR} \quad ax + b \geq c \]

where \( a \neq 0 \), and \( a, b, c \in \mathbb{R} \).

Linear inequalities are solved exactly the same way that linear equations are solved with one exception.

Recall: When solving a linear equation, we can:

1. Add any real number to both sides of the equation.
2. Multiply both sides of an equation by any nonzero real number.

If we look at a small portion of the number line, we see that the integers (\( \mathbb{Z} \)) as well as the reals (\( \mathbb{R} \)) have a natural ordering:

\[ -2 < -1 < 0 < 1 < 2 \]

However, when we multiply every number by \(-1\),
Ex 1

Each inequality becomes false. To correct the situation, we must flip the direction of each inequality:

\[ 2 > 1 > 0 > -1 > -2 \]

Notice that if we add a negative or positive number to each number we just shift the number line, but the original inequalities will still be true:

Ex 2

\[ -3x - 2 < -1 < 6 < 1 < 2 < 3 \]

\[ -5 < x < -3 < -2 < -1 < 0 < 1 \]

If we multiply by a positive number, the original inequalities still hold: (we're stretching the number line)

Ex 3

\[ -2 < -1 < 0 < 1 < 2 \]

\[ -6 < -3 < 0 < 3 < 6 \]

Even positive fractions maintain the original inequalities:

Ex 4

\[ -2 < -1 < 0 < 1 < 2 \]

\[ -1 < -\frac{1}{2} < 0 < \frac{1}{2} < 1 \]

contraction
when solving a linear inequality, we can:

1. Add any real number to both sides of the inequality.
2. Multiply both sides of an inequality by a positive real number.
3. Multiply both sides of an inequality by a negative real number and flip the inequality.

Ex. Solve and graph the solution of:

\[
\begin{align*}
4 - 6x &< 2 & \text{OR} & & 4 - 6x &< 2 \\
-2 & & -2 & & -4 & & -4
\end{align*}
\]

\[
\begin{align*}
2 - 6x &< 0 & & \frac{-6x}{-6} &< \frac{-2}{-6} \\
+6x & \quad +6x & & x &> \frac{1}{3}
\end{align*}
\]

Notice in this solution, since we divided by -6 or multiplied by \(-\frac{1}{6}\), we had to flip the inequality.

Solution graph:

open circle means we don't include this point in the solution set.
Ex. (Problem 52)

Five more than four times a number is at least 21. What range of values can this number have?

Solution: Let \( x \) represent the unknown number(s).

\[
5 + 4x \geq 21
\]

\[
\frac{4x}{4} \geq \frac{16}{4}
\]

\[
x \geq 4
\]

Ex. (Problem 61)

If the revenue function is \( R(x) = 40x \) and the cost function is \( C(x) = 20x + 1600 \), how many items must be sold to realize a profit?
Ex. (problem 69)

You are offered a job with two options:

(a) A salary of $45,000 per year OR
(b) A salary of $2,500 per month plus a commission of 6% on gross sales.

For what range of salaries is the second plan better than the first?