Math 1090 Practice Exam 2 November 22, 2010

Name: SOLUTIONS

Instructions: Answer the questions in the spaces provided on the question sheets. If you need more space, use the bottom of the last page. Partial credit will be awarded. Scientific calculators are allowed, but graphing calculators are not allowed. This exam is closed book and closed notes.

1. Find the domain of the following function.

$$g(x) = \frac{\sqrt{x+5}}{\sqrt{5-x}}$$

$x+5 \geq 0 \Rightarrow x \geq -5$

$5-x > 0 \Rightarrow x < 5$

$$\boxed{-5 \leq x < 5}$$

2. If $f(x) = x^2 + 2$, compute:

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+2-(x^2+2)}{h} = \frac{\cancel{x^2}+2hx+h^2+\cancel{2}-\cancel{x^2}-\cancel{2}}{h}$$

$$= \frac{2hx+h^2}{h} = \boxed{2x+h}$$

3. Let $f(x) = 3x^2$, and let $g(x) = 5x - 1$. Compute $f \circ g(x)$ and $g \circ f(x)$.

$f \circ g(x) = f(g(x)) = f(5x-1) = 3(5x-1)^2$

$g \circ f(x) = g(f(x)) = g(3x^2) = 5(3x^2)-1$

4. Solve the following quadratic equations using the indicated solution methods.

(a) Factor: $-3x^2 - 6x + 9 = 0$ $\Rightarrow$ $x^2 + 2x - 3 = 0$

divide both sides of eqn. by -3

$(x+3)(x-1) = 0$

$\boxed{x = -3, 1}$

(b) Complete the square, and then use the square root technique.

$$-2x^2 + 16x - 11 = 0$$

$-2(x^2 - 8x) - 11 = 0$

$-2(x^2 - 8x + 16 - 16) - 11 = 0$

$-2((x-4)^2 - 16) - 11 = 0$

$-2(x-4)^2 + 32 - 11 = 0$

$-2(x-4)^2 = -21$

$(x-4)^2 = \frac{21}{2}$

$x - 4 = \pm\sqrt{\frac{21}{2}}$

$x = 4 \pm \sqrt{\frac{21}{2}}$

5. True or False.

(a) $(-3, 4)$ is the vertex of the parabolic function $y = 2(x+3)^2 - 4$.

A. True B. False

$(h, k) = (-3, -4)$

(b) The graph of an even degree polynomial always has at least two real roots.

A. True B. False

(c) The graph of an odd degree polynomial always has at least one real root.

A. True B. False

(d) A rational function may have several vertical asymptotes.

A. True B. False

(e) A rational function may have several horizontal asymptotes.

A. True B. False

(f) The graph of a rational function never intersects a horizontal asymptote.

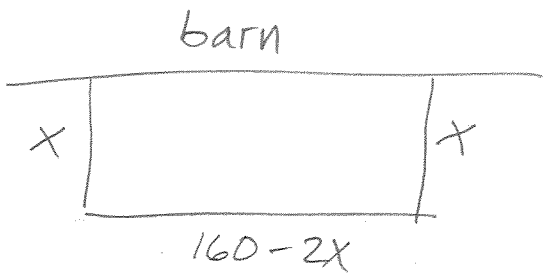
A. True B. False

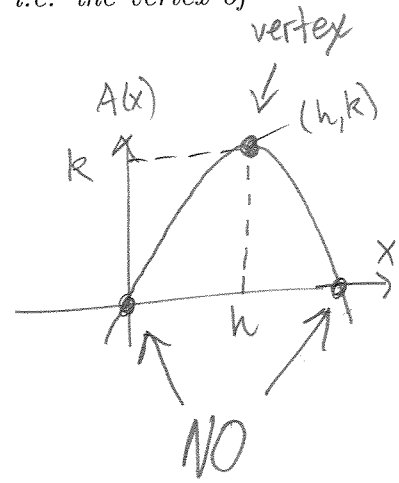
(g) The logarithmic expression $\log_1 2$ is equivalent to some real number.

A. True B. False

6. A farmer wishes to build a rectangular corral for some livestock. He has 160 feet of fencing. The corral will use the side of a barn for one of its sides, the other three sides will be fencing. What dimensions should the corral have in order to ***maximize*** its area?

Hint: Start with an equation for the perimeter, combine that with the area function to create a quadratic equation in one variable, then find the maximum, i.e. the vertex of the parabola.





$$A = x \cdot (160 - 2x) = -2x^2 + 160x$$

Now <u>complete the square</u> to put the area function A into <u>vertex form</u>, so we can find the highest point on the parabola, i.e. <u>maximize the area</u>.

$$A(x) = -2(x^2 - 80x)$$
$$= -2(x^2 - 80x + 1600 - 1600)$$
$$= -2((x - 40)^2 - 1600)$$
$$= -2(x - 40)^2 + 3200$$

$\Rightarrow (h, k) = (40, 3200) \Rightarrow$ dimensions $= 40 \times (160 - 2 \cdot 40)$

$$\boxed{\text{dimensions} = 40 \times 80}$$

7. Analyze and graph the rational function $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 4}$. $= \dfrac{(x+5)(x-3)}{(x+2)(x-2)}$

(a) Find the vertical asymptote(s).

$x = -2$, $x = 2$

(b) Find the horizontal asymptote.

$y = 1$

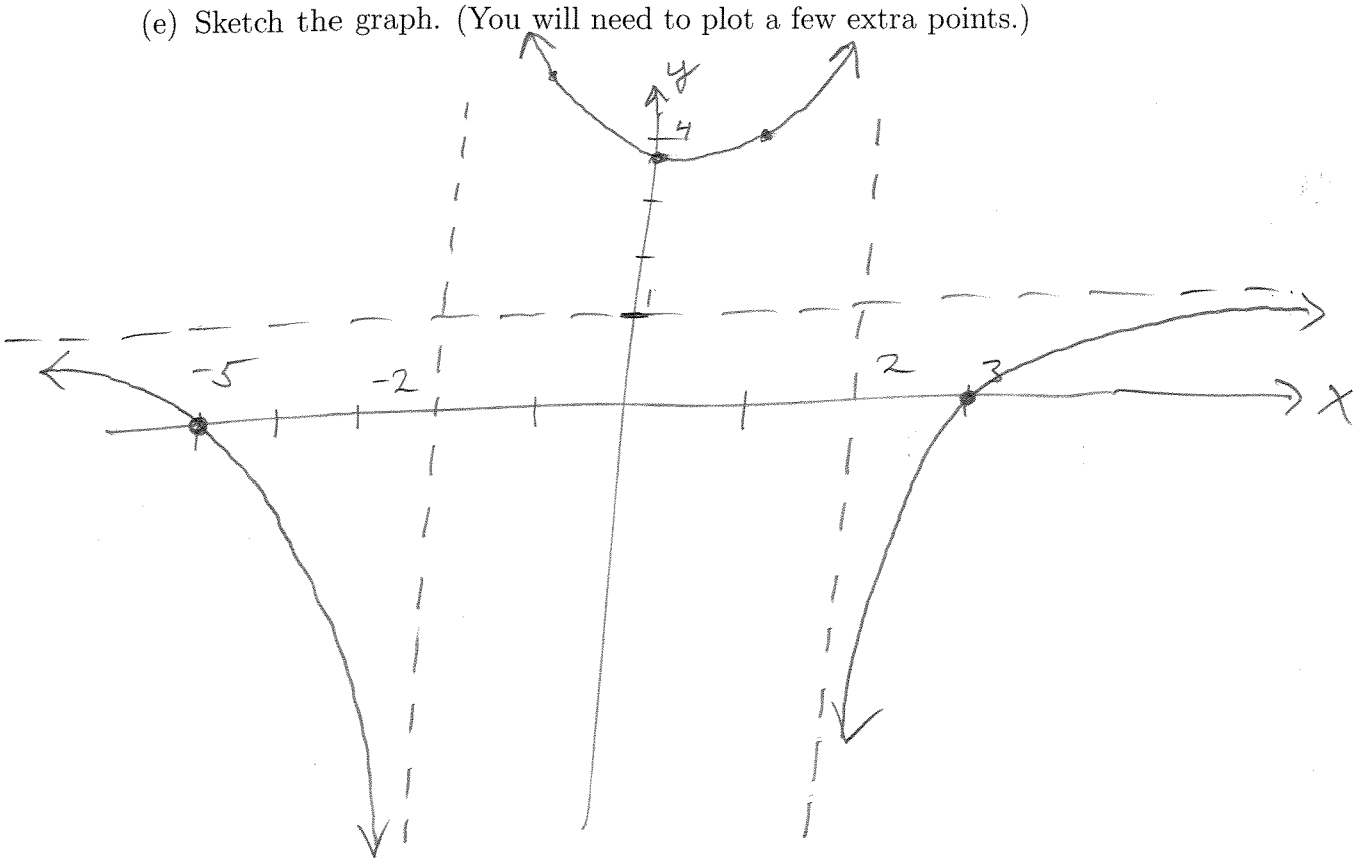
(c) Find the y-intercept.

$y = \dfrac{-15}{-4} = \dfrac{15}{4} = 3\dfrac{3}{4}$

(d) Find the x-intercept(s), i.e. zeros of the function.

$x = -5, 3$

(e) Sketch the graph. (You will need to plot a few extra points.)



$f(1) = \dfrac{1+2-15}{1-4} = \dfrac{-12}{-3} = 4$

$f(-1) = \dfrac{1-2-15}{1-4} = \dfrac{-16}{-3} = 5\dfrac{1}{3}$

8. Let $f(x) = 2(x-4)^3 + 1$. We can obtain the graph of $f(x)$ by shifting, stretching/shrinking and reflecting the base function $b(x) = x^3$, in the horizontal and vertical directions.

(a) What is the vertical shift (amount and direction)? $+1$ $\uparrow$

(b) What is the horizontal shift (amount and direction)? $4 \rightarrow$

(c) Is the graph stretched/shrunk in the vertical direction? If so, by what factor?

Yes, stretched by a factor of 2.

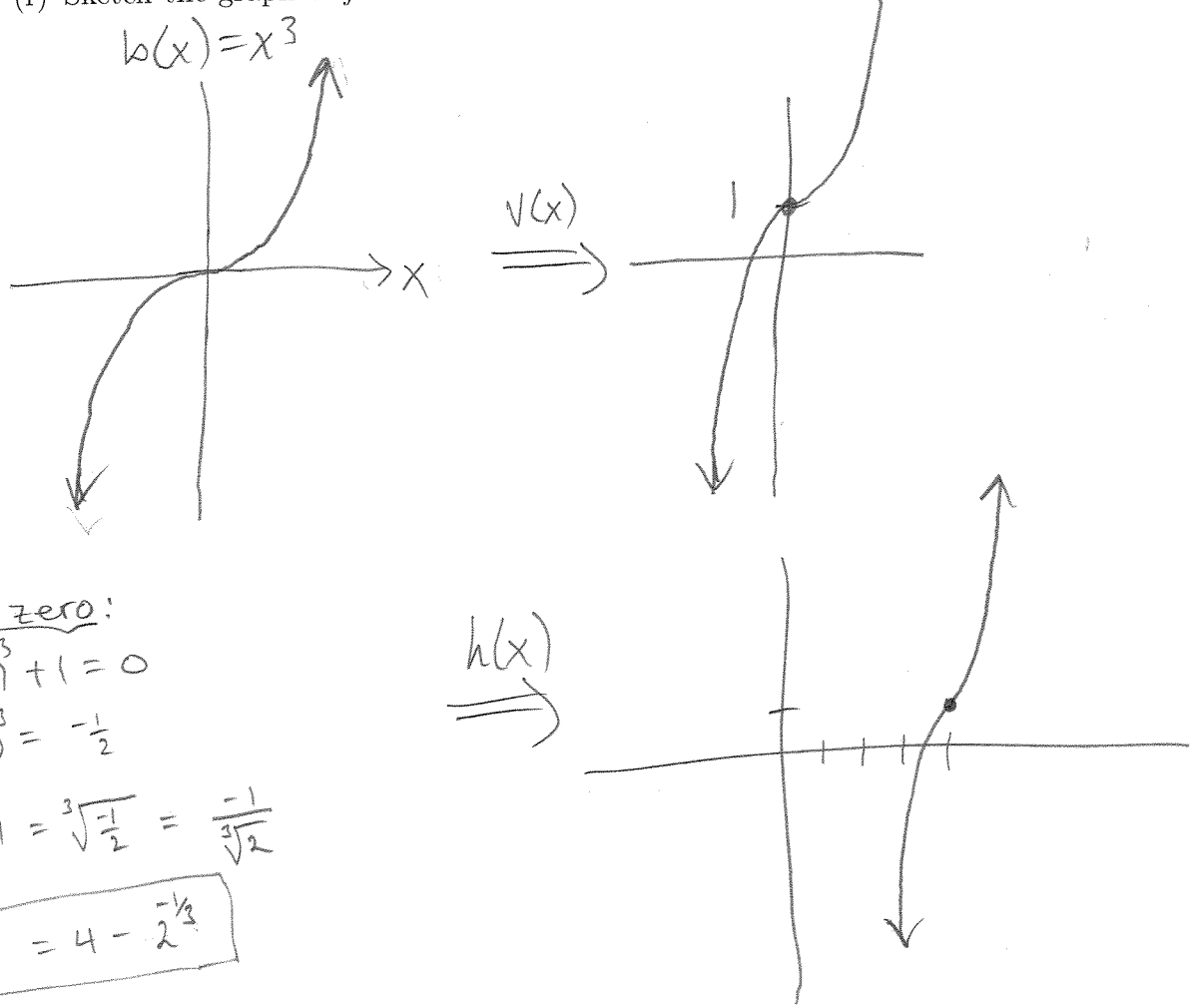
(d) Is the graph stretched/shrunk in the horizontal direction? If so, by what factor?

No

(e) Is the graph reflected? If so, is it a vertical or horizontal reflection?

No reflections

(f) Sketch the graph of f.



Find the zero:

$2(x-4)^3 + 1 = 0$

$(x-4)^3 = -\frac{1}{2}$

$x - 4 = \sqrt[3]{-\frac{1}{2}} = \frac{-1}{\sqrt[3]{2}}$

$\boxed{x = 4 - 2^{-1/3}}$

9. Find the inverse, $f^{-1}(x)$, of the function below.

$$f(x) = \frac{5}{x} - 1$$

Check your work by showing $f \circ f^{-1}(x) = x$ and $f^{-1} \circ f(x) = x$.

$y = \frac{5}{x} - 1 \implies x = \frac{5}{y} - 1$ Now solve for y:

$x + 1 = \frac{5}{y} \implies y(x+1) = 5 \implies y = \boxed{\frac{5}{x+1} = f^{-1}(x)}$

Check: $f \circ f^{-1}(x) = f\left(\frac{5}{x+1}\right) = \frac{5}{\frac{5}{x+1}} - 1 = \frac{\cancel{5}}{1} \cdot \frac{x+1}{\cancel{5}} - 1 = x$ ✓

$f^{-1} \circ f(x) = f^{-1}\left(\frac{5}{x} - 1\right) = \frac{5}{\left(\frac{5}{x} \cancel{-1}\right) \cancel{+1}} = \frac{5}{\frac{5}{x}} = \frac{5}{1} \cdot \frac{x}{5} = x$ ✓

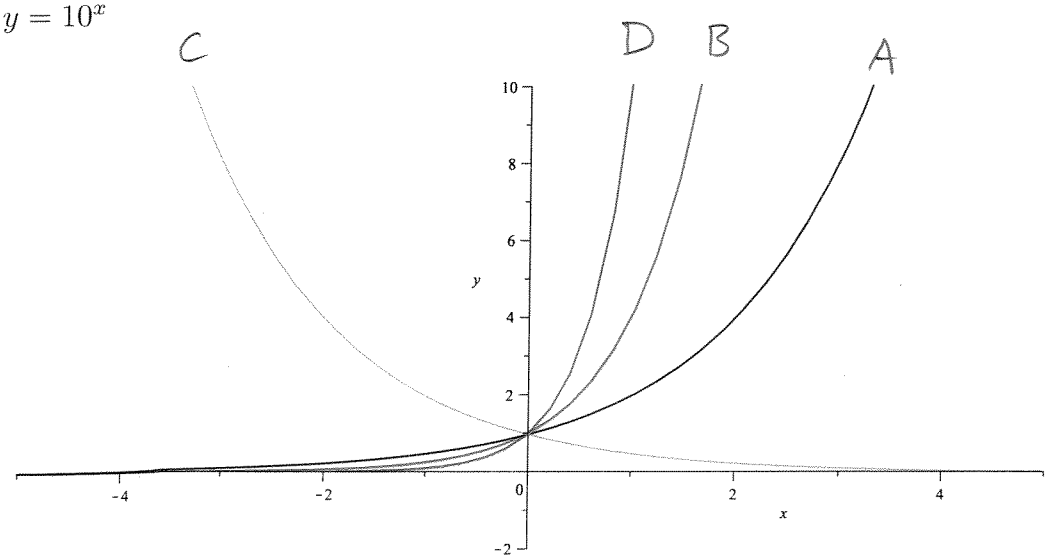
10. Match each function below to its graph by labeling the corresponding graphs with the letters **a**, **b**, **c** and **d**.

(a) $y = 2^x$

(b) $y = 4^x$

(c) $y = \left(\frac{1}{2}\right)^x$

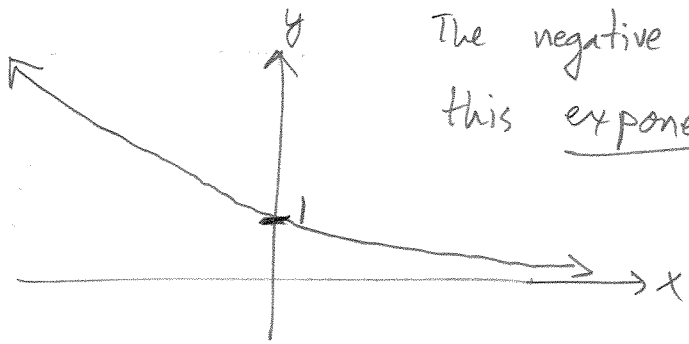
(d) $y = 10^x$



11. Simplify the following expression.

$$\frac{e^{2x}}{e^{1-x}} = e^{2x} \cdot e^{-(1-x)} = e^{2x-(1-x)} = e^{2x-1+x} = \boxed{e^{3x-1}}$$

12. Sketch the graph of the function $y = e^{-x}$.



The negative sign makes this exponential decay.

13. Using the definition of logarithm, rewrite the following logarithm as an exponential, and vice versa.

Recall $a^y = x \iff y = \log_a(x)$

(a) $\log_2 32 = 5$

$2^5 = 32$

(b) $81^{\frac{1}{4}} = 3$

$\log_{81}(3) = \frac{1}{4}$

14. What is the domain of the function $f(x) = \ln(3 - x)$?

$3 - x > 0 \iff 3 > x \iff \boxed{x < 3}$

15. Use the properties of logarithms to expand the expression completely.

$$\log\left[\frac{(x+2)^2}{x-1}\right] = \log\left[(x+2)^2\right] - \log\left[x-1\right]$$

$$= 2\log\left[x+2\right] - \log\left[x-1\right]$$

16. Solve the equation (if possible): $\log_7(2x+3) = \log_7 x - \log_7 2$

$$\left[\log_7(2x+3) - \log_7(x)\right] + \log_7(2) = 0$$

$$\log_7\left(\frac{2x+3}{x}\right) + \log_7(2) = 0$$

$$\log_7\left(\frac{2(2x+3)}{x}\right) = 0$$

$$7^{\log_7\left(\frac{2(2x+3)}{x}\right)} = 7^0 = 1$$

$$\frac{2(2x+3)}{x} = 1$$

$$2(2x+3) = x$$

$$4x+6 = x$$

$$3x = -6$$

$$\boxed{x = -2}$$

Check if $x=-2$ is in domain!

$2x+3>0$ AND $x>0$ but $-2 \not> 0$

$\Rightarrow$ $\boxed{\text{No solution}}$

17. Solve the equation (if possible): $x^2e^x - 5xe^x = 0$

$$e^x(x^2-5x) = 0$$

$$x \cdot e^x(x-5) = 0$$

Since e^x is never 0,

Either $\boxed{x=0 \text{ OR } x=5}$

18. Use the Change of Base formula given below to rewrite the following expression using natural logarithms (i.e. ln).

$$\boxed{\log_b x = \frac{\log_a x}{\log_a b}}$$

$$\log_{\left(\frac{3}{7}\right)} 4 = \frac{\ln(4)}{\ln\left(\frac{3}{7}\right)}$$

19. At the beginning of this year, there were two devestating earthquakes in the news, one in Haiti and one in Chile. Although the earthquake in Haiti was less powerful, more people were killed. According to the BBC News web site:

> The Haiti quake measured 7.0 and because the epicentre was so close to the ill prepared capital, Port-au-Prince, the damage was severe, and over 200,000 people died as a result.
>
> The death toll in Haiti is in stark contrast to the magnitude 8.8 earthquake that struck Chile in February 2010 where less than 1,000 people died.

Using Richter's formula for the magnitude of an earthquake below, determine how much more energetic the Chilean quake was compared to the Haitian earthquake.

$$M = \frac{2}{3}\log\left(\frac{I}{I_0}\right).$$

In other words, compute $\frac{I_C}{I_H}$, where I_C represents the intensity of the Chilean earthquake, and I_H represents the intensity of the Haitian earthquake.

$$M_C = 8.8 = \frac{2}{3}\log\left(\frac{I_C}{I_0}\right) \qquad M_H = 7.0 = \frac{2}{3}\log\left(\frac{I_H}{I_0}\right)$$

$$M_C - M_H = 8.8 - 7.0 = \frac{2}{3}\log\left(\frac{I_C}{I_0}\right) - \frac{2}{3}\log\left(\frac{I_H}{I_0}\right)$$

$$\Rightarrow \quad 1.8 = \frac{2}{3}\left[\log\left(\frac{I_C}{I_0}\right) - \log\left(\frac{I_H}{I_0}\right)\right]$$

$$\frac{3}{2}(1.8) = \log\left[\frac{\left(\frac{I_C}{I_0}\right)}{\left(\frac{I_H}{I_0}\right)}\right] = \log\left[\frac{I_C}{\cancel{I_0}} \cdot \frac{\cancel{I_0}}{I_H}\right]$$

$$10^{\frac{3}{2}(1.8)} = \cancel{10}^{\cancel{\log}\left(\frac{I_C}{I_H}\right)} = \frac{I_C}{I_H}$$

$$\Rightarrow \boxed{\frac{I_C}{I_H} = 10^{2.7} \approx 501}$$

Thus the Chilean earthquake was about 500 times stronger (more intense) than the Haitian earthquake.

20. In problem 2, you were given $f(x) = x^2 + 2$ and were asked to compute

$$\frac{f(x+h) - f(x)}{h}$$

Simplify this expression as much as possible and then compute:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Your answer should not have any h's in it. If you perform the algebraic simplifications correctly, and get the correct limit, then you just solved a Calculus problem. The above expression is the derivative of $f(x)$ denoted $f'(x)$.

$$\frac{f(x+h) - f(x)}{h} = \frac{((x+h)^2 + 2) - (x^2 + 2)}{h}$$

$$= \frac{\cancel{x^2} + 2hx + h^2 + \cancel{2} - \cancel{x^2} - \cancel{2}}{h}$$

$$= \frac{2hx + h^2}{h} = \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \boxed{2x + h}$$

$$\Rightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 2x + h = 2x$$

$$\Rightarrow \boxed{f'(x) = 2x}$$