Name: ______________________________

Instructions: Answer the questions in the spaces provided on the question sheets. If you need more space, use the bottom of the last page. Partial credit will be awarded. Basic calculators are allowed, but graphing calculators are not allowed. This exam is closed book and closed notes.

1. [5 points] Solve the linear inequality *and graph the solution* on a number line.

$$-2x + 5 \leq -9$$

2. [10 points] If \$12 is worth 1000 ¥, and \$60 is worth 5000 ¥. Then how much is \$33 worth in yen?

   Hint: Let $x =$ dollars, $y =$ yen, plot the two points on a graph. Determine the equation of a line passing through the two points.

3. You are considering buying a hot dog cart and selling hot dogs at the stadium during football games. The cart costs \$1000. You will also need to pay a one time fee of \$500 to get a vendor license. It costs you \$0.50 per hot dog and \$0.15 per bun. (You may neglect the cost of condiments.) You plan to sell your hot dogs for \$3.00 each.

   (a) [3 points] Write the revenue function $R(x)$.

   (b) [3 points] Write the cost function $C(x)$.

   (c) [3 points] Determine the profit function $P(x)$.

   (d) [3 points] What is the marginal profit for this product?

   (e) [3 points] How many hot dogs must you sell to break even? Round your answer to the nearest hot dog.

4. A factory can manufacture up to 1,000 vehicles (cars and trucks) per day. The factory must manufacture at least 200 more cars than trucks. The profit from selling a car is $1,000, and the profit from selling a truck is $1,500.

   Let $x =$ the number of cars, and let $y =$ the number of trucks.

   (a) [5 points] What is the profit/objective function they wish to maximize?

   (b) [5 points] What are the constraints they must satisfy? (Hint: there are 4)

5. Solve the following linear programming problem, by following these steps:

   (a) [8 points] Use the constraints to graphically determine the feasible region.

   (b) [8 points] Find the vertices of the feasible region.

   (c) [4 points] Determine which vertex maximizes the objective function.

   Objective function: $p(x, y) = x + \frac{1}{2}y$

$$\begin{aligned} x + y &\leq 8 \\ -3x + y &\leq 0 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

6. Let A, B and C be $n \times n$ matrices, and let I be the $n \times n$ identity matrix (the matrix with 1s on the diagonal and 0s everywhere else).

   (a) [2 points] The key to solving algebraic equations is to understand associativity, the identity, and inverses.
   A. true B. false

   (b) [2 points] $(AB)C = A(BC)$
   A. always B. sometimes C. never

   (c) [2 points] $AI = A = IA$
   A. always B. sometimes C. never

   (d) [2 points] If A^{-1} exists, then $A^{-1}A = I$
   A. always B. sometimes C. never

   (e) [2 points] $AB = BA$
   A. always B. sometimes C. never

7. (a) [10 points] Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$. Find A^{-1}, using the technique of your choice.

   (b) [10 points] Let $B = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 0 & 9 \end{bmatrix}$, compute AB.

8. [10 points] Solve the following system of equations using Gauss-Jordan elimination on the augmented matrix.

$$
\begin{aligned}
x + 0y - 5z &= 2 \\
2x + y + 0z &= 15 \\
0x + 2y - 6z &= -4
\end{aligned}
$$

9. [5 points (bonus)] $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$, compute AB.

Question	Points	Score
1	5	
2	10	
3	15	
4	10	
5	20	
6	10	
7	20	
8	10	
9	0	
Total:	100	