1. [5 points] Solve the linear inequality and graph the solution on a number line.

\[-2x + 5 \leq -9\]

2. [10 points] If $12 is worth 1000 ¥, and $60 is worth 5000 ¥. Then how much is $33 worth in yen?

Hint: Let \( x = \) dollars, \( y = \) yen, plot the two points on a graph. Determine the equation of a line passing through the two points.
You are considering buying a hot dog cart and selling hot dogs at the stadium during football games. The cart costs $1000. You will also need to pay a one time fee of $500 to get a vendor license. It costs you $0.50 per hot dog and $0.15 per bun. (You may neglect the cost of condiments.) You plan to sell your hot dogs for $3.00 each.

(a) [3 points] Write the revenue function \( R(x) \).

(b) [3 points] Write the cost function \( C(x) \).

(c) [3 points] Determine the profit function \( P(x) \).

(d) [3 points] What is the marginal profit for this product?

(e) [3 points] How many hot dogs must you sell to break even? Round your answer to the nearest hot dog.
4. A factory can manufacture up to 1,000 vehicles (cars and trucks) per day. The factory must manufacture at least 200 more cars than trucks. The profit from selling a car is $1,000, and the profit from selling a truck is $1,500.

Let $x =$ the number of cars, and let $y =$ the number of trucks.

(a) [5 points] What is the profit/objective function they wish to maximize?

(b) [5 points] What are the constraints they must satisfy? (Hint: there are 4)

5. Solve the following linear programming problem, by following these steps:

(a) [8 points] Use the constraints to graphically determine the feasible region.

(b) [8 points] Find the vertices of the feasible region.

(c) [4 points] Determine which vertex maximizes the objective function.

Objective function: $p(x, y) = x + \frac{1}{2}y$

\begin{align*}
    x + y &\leq 8 \\
    -3x + y &\leq 0 \\
    x &\geq 0 \\
    y &\geq 0
\end{align*}
6. Let $A, B$ and $C$ be $n \times n$ matrices, and let $I$ be the $n \times n$ identity matrix (the matrix with 1s on the diagonal and 0s everywhere else).

(a) [2 points] The key to solving algebraic equations is to understand associativity, the identity, and inverses.
   A. true  B. false

(b) [2 points] $(AB)C = A(BC)$
   A. always  B. sometimes  C. never

(c) [2 points] $AI = A = IA$
   A. always  B. sometimes  C. never

(d) [2 points] If $A^{-1}$ exists, then $A^{-1}A = I$
   A. always  B. sometimes  C. never

(e) [2 points] $AB = BA$
   A. always  B. sometimes  C. never

7. (a) [10 points] Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$. Find $A^{-1}$, using the technique of your choice.

(b) [10 points] Let $B = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 0 & 9 \end{bmatrix}$, compute $AB$. 
8. [10 points] Solve the following system of equations using Gauss-Jordan elimination on the augmented matrix.

\[
\begin{align*}
x + 0y - 5z &= 2 \\
2x + y + 0z &= 15 \\
0x + 2y - 6z &= -4
\end{align*}
\]
9. [5 points (bonus)] \( A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} \), compute \( AB \).