3.1

Solve these equations.

(1) \(4x^2 - 25 = 0\)

(2) \(2x^2 + 5x = 12\)

(3) \(x^2 + 4x - 7 = 0\)

(4) \(6x^2 - 5 = 13x\)

(5) \(2(w - 1)^2 - 8 = 0\)

(6) \(x + 5 = \frac{24}{x}\)

(7) \(\frac{2}{x - 5} - \frac{1}{x + 1} = \frac{1}{4}\)

(8) \(2(y + 1)^2 - 14 = y^2 + 5y + 30\)

3.2

Indicate whether or not the relation represents a function. If it is a function, find the domain.

(9) \(3x^2 + 4y^2 = 1\)

(10) \(y = \frac{2x}{(x - 5)(2x + 1)}\)

(11) \(y = (3 + x^2)\sqrt{x + 5}\)

(12) \(y = \frac{18}{\sqrt{x - 4}}\)

Evaluate each function at the given input values.

(13) \(f(x) = 2x^2 - 4x + 1\)
    (a) \(f(3)\)
    (b) \(f(h)\)
(14) \( g(p) = \frac{9}{\sqrt{3p+2}} \)

(a) \( g(0) \)
(b) \( g(w + 1) \)

(15) \( h(x) = \frac{2x+1}{x-8} \)

(a) \( h(4) \)
(b) \( h(x^2) \)

(16) \( f(y) = \sqrt{2 - y + \sqrt{y+1}} \)

(a) \( f(2) \)
(b) \( f(y + h) \)

(17) \( y(x) = \sqrt{x^2 - 4} \)

(a) \( y(3) \)
(b) \( y(x^2 + 1) \)

### 3.3

For each parabola, find the following:
(a) its vertex,
(b) the axis of symmetry,
(c) whether the parabola is concave up or down,
(d) whether the vertex is a maximum or minimum point,
(e) the x-intercept(s), if any exist,
(f) sketch the graph.

(18) \( y = 2(x + 3)^2 - 4 \)

(19) \( y = -(x - 5)^2 + 2 \)

(20) \( y = (x - 2)^2 + 1 \)

(21) \( y = -\frac{1}{2} x^2 + \frac{3}{2} x + 5 \)

(22) \( y = -4(x + 1)^2 - 1 \)

(23) \( y = x^2 - 6x + 5 \)

(24) \( y = \frac{2}{9} x^2 - 2 \)
(25) A farmer wants to fence a rectangular field and then divide it in half with a fence down the middle parallel to one side. If 1488 feet of fence is to be used, what is the maximum area of the overall lot?

(26) If a profit function is given by  
\[ P(x) = -2x^2 + 2320x - 1000 \]  
where \( x \) represents the number of motor bikes made and sold, how many motor bikes need to be sold for maximum profit? What is the maximum profit?

3.4

Find the number of units that need to be produced and sold to break even, given the revenue and cost functions.

(27) revenue:  
\[ R(x) = 1600x - x^2 \]
  cost:  
\[ C(x) = 1.6(x - 500)^2 + 200000 \]

(28) revenue:  
\[ R(x) = -0.4x^2 + 53.6x \]
  cost:  
\[ C(x) = 0.4x^2 + 12.8x + 496 \]

(29) revenue:  
\[ R(x) = -0.9x^2 + 73x \]
  cost:  
\[ C(x) = 0.6x^2 + 10x + 588 \]

Given the supply and demand equations, find the equilibrium quantity and price.

(30) supply:  
\[ p = q^2 + 8q + 20 \]
  demand:  
\[ p = 100 - 4q - q^2 \]

(31) supply:  
\[ p = 2(q + 5)^2 + 6 \]
  demand:  
\[ p = -7(q + 3)^2 + 994 \]

(32) supply:  
\[ 5p - q = 18 \]
  demand:  
\[ pq = 3540 + 19q \]

(33) The supply and demand functions for Margo's design products are given by  
\[ \frac{p}{q} = 2 \]
\[ pq = 10(20 + 4) \]  respectively. If Margo is taxed $4 per product and passes that tax on to the consumer as a price increase, find the market equilibrium point.

(34) The cost of producing Bill's handiwork is given by  
\[ C(x) = 0.4x^2 + 12.8x + 496 \]  and he's found the demand for his product obeys the equation  
\[ p = -0.4x + 53.6 \]  . If he can only produce fewer than 30 of his handiwork projects per week, how many must he make and sell to break even? How much profit will he make if he sells all 30 units?
3.5

For each polynomial function, find/do the following:
(a) its degree,
(b) all the zeros (roots),
(c) the y-intercept,
(d) the x-intercept(s), if any exist,
(e) sketch the graph.

(35) \( y = \frac{1}{5}(x^3 + 2x^2 - 8x) \)

(36) \( y = \frac{1}{8}(x^2 - 2x - 3)^2 \)

(37) \( y = -\frac{1}{16}(x + 4)(x^2 - 16) \)

(38) \( y = -\frac{1}{24}(x - 2)^3((x - 2)^2 - 16) \)

(39) \( y = x^5 - 2x^3 + x \)

For each piecewise function, fill in the table of points and then sketch the graph.

(40) \( f(x) = \begin{cases} 
3x - 1, & x \geq 0 \\
-x^2 + 2, & x < 0 
\end{cases} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(41) \( y(x) = \begin{cases} 
\frac{1}{2}x^2, & x \geq 2 \\
x, & -1 \leq x < 2 \\
-2x - 2, & x < -1 
\end{cases} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(42) \( h(x) = \begin{cases} 
8, & x \geq 2 \\
x^3, & -2 \leq x < 2 \\
-1, & x < -2 
\end{cases} \)

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.6

Analyze each rational function by answering the following questions.
(a) What is the domain?
(b) Find the vertical asymptote(s), if there are any.
(c) Find the horizontal asymptote, if it exists.
(d) Find the y-intercept, if it exists.
(e) Find the x-intercept(s), if any exist.
(f) Sketch the graph.

(43) \( f(x) = \frac{3}{x-4} \)

(44) \( y=\frac{2x+1}{x} \)

(45) \( y=\frac{x^2-4}{x^2-x-2} \)

(46) \( y=\frac{x}{(x+2)(x-2)} \)

(47) \( y=\frac{4}{x^2-9} \)

3.7

For each function, answer the following questions, compared to the base function graph.
(a) What is the vertical shift?
(b) What is the horizontal shift?
(c) Is the graph stretched/shrunk vertically? If so, by what factor?
(d) Is the graph stretched/shrunk horizontally? If so, by what factor?
(e) Is the graph reflected? If so, is it a vertical or horizontal reflection?
(f) Sketch the graph.

(48) \( y=-|x+1|-3 \)

(49) \( f(x)=\frac{1}{2}(x+2)^3+5 \)

(50) \( h(x)=\sqrt{-4(x-1)}+2 \)

(51) \( g(x)=-x^2-6x+1 \)

(52) \( y=2(x-4)^3 \)
For each pair of given functions, find the following:

(a) \((f + g)(x)\)
(b) \((fg)(1)\)
(c) \((f \circ g)(x)\)
(d) \(g(f(x))\)
(e) \((g - f)(2)\)
(f) \(\left(\frac{f}{g}\right)(3)\)
(g) \((f \circ f)(x)\)

(53) \(f(x) = \sqrt{2x+1}\)
\(g(x) = -x + 5\)

(54) \(f(x) = x^2 + 3\)
\(g(x) = |x| - 1\)

(55) \(f(x) = x^3 + x\)
\(g(x) = \frac{2}{x+1}\)

(56) \(f(x) = \frac{x+5}{x-3}\)
\(g(x) = x^2\)

(57) \(f(x) = 2x - 6\)
\(g(x) = \frac{x}{x^2 - 25}\)

(58) \(f(x) = \sqrt{x}\)
\(g(x) = x^3 - 1\)

(59) \(f(x) = -4x\)
\(g(x) = x^2 + 3x\)
Chapter 3 Review Answer Key

(1) \( x = \frac{\pm 5}{2} \)

(2) \( x = -4 , \frac{3}{2} \)

(3) \( x = -2 \pm \sqrt{11} \)

(4) \( x = -\frac{1}{3} , \frac{5}{2} \)

(5) \( w = -1 , 3 \)

(6) \( x = 3 , -8 \)

(7) \( x = 4 \pm 3\sqrt{5} \)

(8) \( y = -6 , 7 \)

(9) not a function

(10) is a function; domain: \( x \in \mathbb{R} , x \neq 5 , -\frac{1}{2} \)

(11) is a function; domain: \( x \in \mathbb{R} , x \geq -5 \)

(12) is a function; domain: \( x \in \mathbb{R} , x > 4 \)

(13) (a) 7 (b) \( 2h^2 - 4h + 1 \)

(14) (a) \( \frac{9}{\sqrt{2}} \) or \( \frac{9\sqrt{2}}{2} \) (b) \( \frac{9}{\sqrt{3}w + 5} \)

(15) (a) \( -\frac{9}{4} \) (b) \( \frac{2x^2 + 1}{x^2 - 8} \)

(16) (a) \( \sqrt{3} \) (b) \( \sqrt{2 - y - h + \sqrt{y + h + 1}} \)

(17) (a) \( \sqrt{5} \) (b) \( \sqrt{x^4 + 2x^2 - 3} \)
(18) (a) (-3, -4) (b) x = -3 (c) up (d) minimum (e) (-3 + \sqrt{2}, 0) and (-3 - \sqrt{2}, 0)

(19) (a) (5, 2) (b) x = 5 (c) down (d) maximum (e) (5 + \sqrt{2}, 0) and (5 - \sqrt{2}, 0)

(20) (a) (2, 1) (b) x = 2 (c) up (d) minimum (e) none

(21) (a) \left(\frac{3}{2}, \frac{49}{8}\right) (b) x = \frac{3}{2} (c) down (d) maximum (e) (-2, 0) and (5, 0)
(22) (a) (-1, -1) (b) x = -1 (c) down (d) maximum (e) none

(23) (a) (3, -4) (b) x = 3 (c) up (d) minimum (e) (1, 0) and (5, 0)

(24) (a) (0, -2) (b) x = 0 (c) up (d) minimum (e) (3, 0) and (-3, 0)

(25) 372 square feet

(26) 580 boxes, $671,800

(27) ~231 or 1000 units

(28) 20 or 31 units

(29) 14 or 28 units

(30) (4, $68)

(31) (7, $294)
(32) (177, §39)

(33) (10, §24)

(34) 20; $8

(35) (a) 3  (b) x = 0, 2, -4  (c) (0, 0)  (d) (0, 0), (2, 0) and (-4, 0)  
    (e) 

(36) (a) 4  (b) x = -1, 3  (c) 0, 9/8  (d) (-1, 0) and (3, 0)  
    (e) 

(37) (a) 3  (b) x = -4, 4  (c) (0, 4)  (d) (4, 0) and (-4, 0)  
    (e)
(38) (a) 4  (b) x = 2, -2, 6  (c) (0, 2)  (d) (-2, 0), (0, 0) and (6, 0)
    (e)

(39) (a) 5  (b) x = 0, 1, -1  (c) (0, 0)  (d) (0, 0), (1, 0) and (-1, 0)
    (e)

(40)

(41)

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-2</td>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>4.5</td>
</tr>
</tbody>
</table>
(42)

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-1</td>
<td>-8</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

(43) (a) $x \in \mathbb{R}, x \neq 4$  (b) $x = 4$  (c) $y = 0$  (d) $(0, -0.75)$  (e) none

(f) \[
\begin{align*}
\text{Additive pts} & \\
(3, -3) & \\
(5, 3) & \\
\left(\frac{3}{2}, \frac{1}{3}\right) & \\
\end{align*}
\]

(44) (a) $x \in \mathbb{R}, x \neq 0$  (b) $x = 0$  (c) $y = 2$  (d) none  (e) $(-0.5, 0)$

(f) \[
\begin{align*}
\text{Additive pts} & \\
(1, 3) & \\
(-1, 1) & \\
(3, \frac{7}{3}) & \\
(-3, \frac{5}{3}) & \\
\end{align*}
\]

(45) (a) $x \in \mathbb{R}, x \neq 2, -1$  (b) $x = -1$  (c) $y = 1$  (d) $(0, 2)$  (e) $(-2, 0)$

(f) \[
\begin{align*}
\text{Additive pts} & \\
(1, \frac{3}{2}) & \\
(-3, \frac{3}{2}) & \\
\text{open dot at } (2, \frac{4}{3}) & \\
\end{align*}
\]
(46) (a) \( x \in \mathbb{R}, x \neq \pm 2 \)  (b) \( x = 2, x = -2 \)  (c) \( y = 0 \)  (d) \( (0, 0) \)  (e) \( (0, 0) \)

(47) (a) \( x \in \mathbb{R}, x \neq \pm 3 \)  (b) \( x = 3, x = -3 \)  (c) \( y = 0 \)  (d) \( \left(0, \frac{-4}{9}\right)\)  (e) none

(48) (a) down 3  (b) left 1  (c) no  (d) no  (e) yes; vertical

(49) (a) up 5  (b) left 2  (c) yes; shrunk by half  (d) no  (e) no
(50) (a) up 2  (b) right 1  (c) no  (d) yes; shrunk 4  (e) yes; horizontal

(51) (a) up 10  (b) left 3  (c) no  (d) no  (e) yes; vertical

(52) (a) none  (b) right 4  (c) yes; stretched 2  (d) no  (e) no

(53) (a) $\sqrt{2x+1}-x+5$  (b) $4\sqrt{3}$  (c) $\sqrt{-2x+11}$  (d) $5-\sqrt{2x+1}$  (e) $-7-\sqrt{5}$

(f) $\frac{\sqrt{7}}{2}$

(g) $\sqrt{2\sqrt{2x+1}+1}$

(54) (a) $x^2+|x|+2$  (b) 0  (c) $x^2-2|x|+4$  (d) $|x^2+3|-1$ or $x^2+2$  (e) -6

(f) 6  (g) $x^4+6x^3+12$

(55) (a) $x^3+x+\frac{2}{x+1}$  (b) 2  (c) $\frac{2(x^2+2x+5)}{(x+1)^3}$  (d) $\frac{2}{x^2+x+1}$  (e) $-\frac{28}{3}$

(f) 60  (g) $x^9+3x^7+3x^5+2x^3+x$
(56) (a) $x^2 + \frac{x+5}{x-3}$ (b) -3 (c) $\frac{x^3+5}{x^2-3}$ (d) $\frac{(x+5)^2}{(x-3)^2}$ (e) 11
(f) DNE (g) $\frac{5-3x}{x-7}$

(57) (a) $2x-6 + \frac{x}{x^2-25}$ (b) $\frac{1}{6}$ (c) $\frac{2x}{x^2-25} - 6$ (d) $\frac{2x-6}{4x^2-24x+11}$ (e) $\frac{40}{21}$
(f) 0 (g) $4x - 18$

(58) (a) $x^3 + \sqrt{x} - 1$ (b) 0 (c) $\sqrt[3]{x^3} - 1$ (d) $x \sqrt[3]{x} - 1$ (e) $7 - \sqrt{2}$
(f) $\frac{\sqrt{2}}{26}$ (g) $\frac{\sqrt{3}}{2 \sqrt{2} \sqrt{3}}$

(59) (a) $x^2 - x$ (b) -16 (c) $-4x^2 - 12x$ (d) $16x^2 - 12x$ (e) 18
(f) $\frac{2}{3}$ (g) $16x$